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Circular Intuitionistic Fuzzy Decision Making and Its Application

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ABSTRACT

Circular intuitionistic fuzzy set (C-IFS) is introduced by Atanassov in 2020 as an extension of intuitionistic fuzzy sets. It is represented by a circle with a radius (*r*) of each element consist of degrees of membership and non-membership. Several MCDM methods based on distance measures of C-IFS are already proposed in the literature. The primary objective of this study is the development, with the use of the C-IFS, of a new formulation of functions to form a novel C-IFS multi-criteria decision making (MCDM) method. In addition to the existing literature, this study contributes to circular intuitionistic fuzzy sets by proposing some formulations on radius calculation and a new defuzzification function for C-IFS. The optimistic and pessimistic points are also defined on the set to identify a novel score function and an accuracy function with decision-makers attitude (λ). When the perspective of the decision-maker (λ) approaches 1, it means that C-IFS is defuzzified close to its optimistic point, and when the perspective (λ) approaches 0, it is defuzzified close to the pessimistic point of C-IFS. With the use of these functions, a novel C-IFS MCDM method is presented based on criteria weighting and alternative ranking algorithms. This technique is applied to a supplier selection problem for a seamless supply chain network. A sensitivity analysis is also performed to test the effect of parameter changes on the final results. The findings of the study are compared with the results of a classical IFS-MCDM model. Since C-IFS is an extension of IFS, in addition to similar rankings, more precise results are obtained by considering the optimistic and pessimistic points by including the decision-maker attitude in the functions proposed for C-IFS. The study is a pioneer in the C-IFS literature by presenting C-IFS defuzzification function and a new C-IFS MCDM procedure.

1. Introduction

Based on classical set theory rules, an element is either an element of a set or it is not (Zimmermann, 2010). In classical logic, a fundamental yes–no dichotomy is mentioned. In fact, there are evaluations that place between these extremes and cannot be quantified according to the classical set approach. "0–1" precision is not adequate for detecting and solving real-life problems. The way people perceive nature and reflect their thoughts requires going beyond this approach. It is challenging to express the uncertainty in the judgments of human beings due to their nature. In order to reflect and overcome such situations, fuzzy set theory was introduced by Zadeh (1965). The fuzzy set theory allows the mathematical analysis of situations involving an imprecise environment and vagueness. According to the ordinary fuzzy set theory, the belonging of the elements to the sets is determined by the membership degrees. In this way, a set can contain elements with various membership values. It has been argued that this is an inadequate approach to expressing uncertainty (Peng & Selvachandran, 2019). This concept has been developed over time and many extensions of fuzzy set theory have been presented, such as neutrosophic fuzzy sets (Smarandache, 1998), hesitant fuzzy sets (Torra, 2010), Pythagorean fuzzy sets (Yager, 2013), q-rung orthopair fuzzy sets (Yager, 2017), spherical fuzzy sets (Kutlu Gündoğdu & Kahraman, 2019), fermatean fuzzy sets (Senapati & Yager, 2020), and so on. The extensions of fuzzy sets chronologically are shown in Table 1.

One of the most used fuzzy sets is IFS, which is established by Atanassov (1986). Unlike classical (ordinary) fuzzy set theory, not only membership degree is included, but non-membership and indeterminacy degree notations also are defined. IFS can be used in problem-solving and decision making by integrating with many tools in order to benefit from their functionality in indicating vagueness and ambiguity. IFS can be combined with MCDM methods such as SIR method Chai and Liu (2010, September), VIKOR (Devi, 2011), MULTIMOORA (Baležentis et al., 2014), PROMETHEE (Liao & Xu, 2014), entropy (Gumus et al., 2016), and TOPSIS (Ren et al., 2017) in decision making cases including many criteria and alternatives. IF-BWM and AHP

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Table 1
The extensions of fuzzy sets (Adapted from Donyatalab et al., 2020).

Fuzzy set	Reference	Main Characteristics
Ordinary fuzzy set	(Zadeh, 1965)	Elements belong to a set with membership degrees.
Type-2 fuzzy set	(Zadeh, 1975)	The membership degrees are expressed as fuzzy numbers.
Interval-valued fuzzy set	(Zadeh, 1975)	Membership degree places inside an interval.
Intuitionistic fuzzy set	(Atanassov, 1986)	The degree of membership and degree of non-membership are defined whose sum is less than or equal to 1.
Fuzzy multiset	(Yager, 1986)	An element of a set can form more than once with different or the same membership degrees.
Neutrosophic fuzzy set	(Smarandache, 1998)	This set contains the truth membership, indeterminacy membership, and falsity membership function.
Nonstationary fuzzy set	(Garibaldi & Ozen, 2007)	It is a type-1 fuzzy set in which has a connection, which is expressed as a variation in the membership function over time, between the membership functions.
Hesitant fuzzy set	(Torra, 2010)	It is used to simulate uncertainty caused by hesitation while assigning membership degrees. The set is described by a set of terms of a function that returns in the domain.
Pythagorean fuzzy set	(Yager, 2013)	The sum of the squares of the degrees of membership and degrees of non-membership is less than or equal to 1.
Picture fuzzy set	(Cường, 2015)	It includes degree of positive, neutral and negative membership notations whose sum is less than or equal to 1.
q-rung orthopair fuzzy set	(Yager, 2017)	The sum of the qth powers of the membership and the non-membership degrees is less than or equal to 1 ($q \geq 1$).
Spherical fuzzy set	(Kutlu Gündoğdu & Kahraman, 2019)	The sum of membership, non-membership and hesitancy degrees is less than or equal to 1.
Circular intuitionistic fuzzy set	(Atanassov, 2020)	It contains a circle with a radius r that centers the membership and non-membership degrees.

(Majumder et al., 2021), interval-valued intuitionistic CoCoSo (Alrasheedi et al., 2021), IF-MAIRCA (Ecer, 2022), interval-valued IF-AHP and WASPAS (Alimohammadlou & Khoshsepehr, 2022) are some of the more recent and various industry applications in the literature.

To enlarge the concept of the IFS, Atanassov expanded the concept and introduced C-IFS in 2020 (Atanassov, 2020). Basically, a C-IFS denotes a circle with a radius r that centers the membership and non-membership degrees of IFS. Likewise intuitionistic fuzzy sets on which it is based, C-IFS can be applied on MCDM methods in many application areas. Çakır et al. (2021, 2021, November, 2022) proposed a new circular intuitionistic fuzzy MCDM methodology and conducted studies on medical waste landfill site selection, health tourism and the examination of businesses in terms of industrial symbiosis. C-IFS has recently started to be combined with existing MCDM models in the literature. Otay and Kahraman (2021) used C-IFS with the AHP and VIKOR methods in a supplier selection problem. Similarly, Kahraman and Alkan (2021) reflected uncertainty by using C-IFS in the implementation of the TOPSIS method. A supplier selection was conducted for fast-moving consumer goods (FMCG), and the findings of the study were compared with the results of the IF-TOPSIS method. In a different study, the authors selected a waste disposal site with a methodology integrating the VIKOR method and C-IFS (Kahraman & Otay, 2021). In another study in the context of site selection, the C-IFS TOPSIS method was employed for site selection for the pandemic hospital (Alkan & Kahraman, 2021).

In order to expand the application areas of C-IFS, which is an extension of IFS, the IFS MCDM cases have been examined. The methods have been applied in miscellaneous areas, such as selecting personnel (Zhang & Liu, 2011), assessing the risk of failure modes (Wang et al., 2016), selecting site for solid waste disposal (Kahraman et al., 2017), evaluating criteria of projects (Khorram & Khalegh, 2020), evaluating healthcare waste disposal technology (Mishra et al., 2020), and appraising processes of bio-energy production (Mishra et al., 2020). Supplier selection and evaluation problems are also suitable for the implementation of intuitionistic fuzzy MCDM methods. Boran et al. (2009) benefited intuitionistic fuzzy TOPSIS to select the best supplier among five alternatives for an automotive company. The same method was also used for facility location evaluation (Boran, 2011) and supplier selection problems (Makui et al., 2016). Krishankumar et al. (2017) provided intuitionistic fuzzy PROMETHEE method to prioritize the alternative suppliers for a manufacturing company. Büyüközkan and Göçer (2017) integrated intuitionistic fuzzy axiomatic design and intuitionistic fuzzy analytic hierarchy process (AHP) methods in order to evaluate sport goods brand suppliers in Turkey. The authors selected the best supplier in a digital supply chain using intuitionistic fuzzy AHP and ARAS methods in another study (Büyüközkan & Göçer, 2018). Sen et al. (2018) compared the results of three intuitionistic MCDM methods for a sustainable supplier selection problem. Similarly, Phochanikorn and Tan (2019) also evaluated sustainable suppliers using DEMATEL based analytic network process and VIKOR methods with IFS. Demircioğlu and Ulukan (2020) applied intuitionistic fuzzy SAW and PROMETHEE methods for the environmental ranking of cities. Green supplier selection is also an eligible problem for intuitionistic fuzzy MCDM methods (Rouyendegh et al., 2020; Kumari & Mishra, 2020). As a step forward, establishing a seamless supply chain is one of the primary organizational goals of business managers (Love et al., 2004). Maintaining uninterrupted information and material flow throughout the supply chain processes ensures high business performance (Özbayrak et al., 2007). All supply chain components in a seamless supply chain need to act as integrated and complementary (Towill, 1997;). Selecting the suppliers with the highest performance and compatibility is critical (Torkayesh et al., 2020; Tirkolaee et al., 2021). The concept can be examined from countless perspectives and its merits have been tried to be revealed. Fuzzy MCDM methodologies are appropriate tools for evaluating suppliers in seamless supply chains.

Considering the rise of fuzzy logic in decision making, this study extends the idea of C-IFS and proposes new functions in detail in order to increase the usage of C-IFS in MCDM methods. It describes novel perspectives in radius calculation and the inclusion of interior points in defuzzification decision-makers attitude. MCDM steps are proposed and illustrated in the example for easy use of C-IFS numbers in decision making cases. Sensitivity analysis is performed according to parameter changes, and the results are compared with IF MCDM procedure. This study is a pioneer in the C-IFS literature by presenting C-IFS defuzzification function and a new C-IFS MCDM procedure.

This article is organized as follows. Section 2 gives the preliminaries of C-IFS and the new perspectives on the interpretation of C-IFS. IFS and IV-IFS are examined, and new score and defuzzification functions are produced gradually for C-IFS numbers based on previous fuzzy sets. Section 3 proposes the algorithm to use C-IFS in MCDM cases in two parts. The criteria weight generation and alternative ranking procedures are given step by step. Section 4 illustrates the C-IFS MCDM approaches in three categories which are deriving criteria weights, ranking the

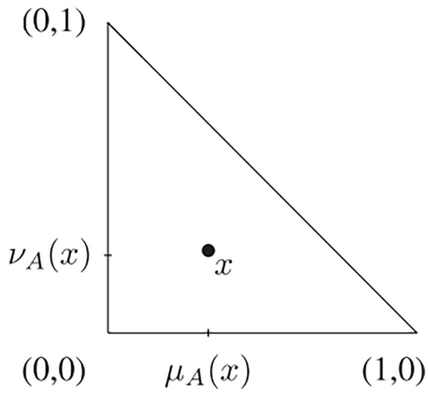

Fig. 1. Geometrical presentation of IFS.

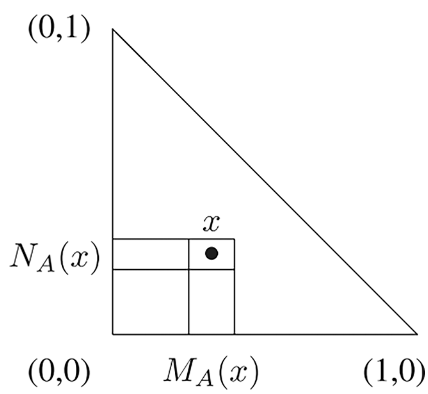

Fig. 2. Geometrical presentation of IVIFS.

alternatives according to the criteria and comparison the results of C-IFS MCDM with the results of IF MCDM procedure to validate the methodology. It performs sensitivity analysis for decision making attitude and compares the results of different radius calculatıon operators. Section 5 gives an overview of the study and discusses the results. In the last section, the study is concluded by highlighting the advantages and limitations of the proposed approaches and recommending future directions on C-IFS.

2. Preliminaries for C-IFS

Circular intuitionistic fuzzy set (C-IFS) is first developed by Atanassov (2020) as an extension of IFS to enlarge the definition for vagueness of expression. It is represented by a circle of each element consist of degrees of membership and non-membership. This section gives the definitions and new ideas on C-IFS.

For the classical definition of IFS in Definition 1, it is evaluated with membership and non-membership degrees in a fixed universe interpreted as points on the intuitionistic fuzzy interpretation triangle (IFIT). The geometric presentation of IFS is in Fig. 1.

Definition 1. Atanassov, 1986 *Let* $X = \{x_1, x_2, \cdots, x_n\}$ *be a universe of discourse, an IFS A in X is given by*

$$A = \{< x, u_A(x), v_A(x) > x \in X\} \quad (1)$$

where "$u_A : X \rightarrow [0,\ 1]$" and "$v_A : X \rightarrow [0,\ 1]$" with the conditions $0 \leq u_A(x) + v_A(x) \leq 1, \forall x \in X$. These numbers are the membership degree "$u_A(x)$" and "$v_A(x)$" non-membership degree of the element $\times$ to the set A, respectively. Given an element $\times$ of X, the pair "$< u_A(x), v_A(x) >$" is called an intuitionistic fuzzy value (IFV) (Lim & Kim, 2005). For convenience, it can be denoted as $\widetilde{a} =< u_{\widetilde{a}}, v_{\widetilde{a}} >$ such that $u_{\widetilde{a}} \in [0,1]$, $v_{\widetilde{a}} \in [0,1]$ and $0 \leq u_{\widetilde{a}} + v_{\widetilde{a}} \leq 1$. The indeterminacy degree is denoted by $\pi_{\widetilde{a}}$, with the conditions of $\pi_{\widetilde{a}} \in [0,1]$ and $\pi_{\widetilde{a}} = 1 - u_{\widetilde{a}} - v_{\widetilde{a}}$.

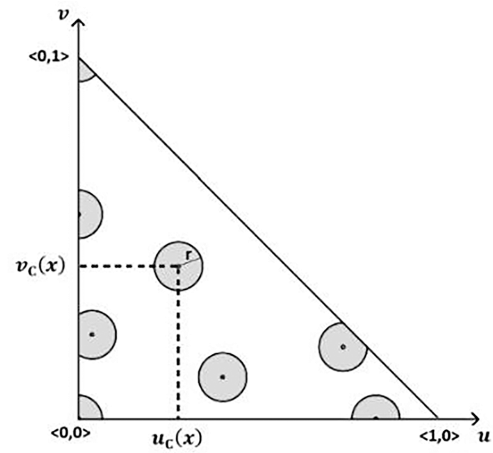

Fig. 3. Geometrical representation of circular intuitionistic fuzzy numbers.

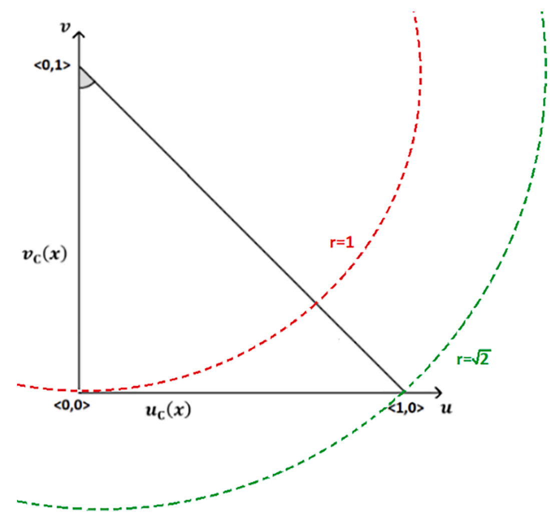

Fig. 4. Triangle coverage of different *r* values of C-IFS.

The complement of an IF $\widetilde{a} =< u_{\widetilde{a}}, v_{\widetilde{a}}, \pi_{\widetilde{a}} >$ is defined as follows:

$$\widetilde{a}^C =< v_{\widetilde{a}}, u_{\widetilde{a}}, \pi_{\widetilde{a}} > \quad (2)$$

As an extension of IFS given in Definition 2, interval-valued IFS (IVIFS), the membership and non-membership degrees are expressed within a range in a fixed universe. The geometrical presentation of IVIFS is in Fig. 2.

Definition 2. *Let D* [0,1] *be the set of all closed subintervals of* [0,1], *an IVIFS A in X is defined as* $A = \{< x, u_A(x), v_A(x) > x \in X\}$ *where* $u_A : X \rightarrow D[0,1]$ *and* "$v_A : X \rightarrow D[0,\ 1]$", *with the condition* "$0 \leq supu_A(x) + supv_A(x) \leq 1,\ x \in X$". *Similarly, the intervals* $u_A(x)$ *and* $v_A(x)$ *denote the degrees of membership and non-membership of* $\times$ *to A, respectively.*

Given any $x \in X$, the couple $< u_A(x), v_A(x) >$ is called an IVIFN (Atanassov & Gargov, 1989). For convenience, this study denotes an IVIFN by $\widetilde{A} = \left([u_{\widetilde{A}}^{-}, u_{\widetilde{A}}^{+}], [v_{\widetilde{A}}^{-}, v_{\widetilde{A}}^{+}]\right)$, where $[u_{\widetilde{A}}^{-}, u_{\widetilde{A}}^{+}] \in D[0,\ 1]$, $[v_{\widetilde{A}}^{-}, v_{\widetilde{A}}^{+}] \in D[0,1]$ and $u_{\widetilde{A}}^{+} + v_{\widetilde{A}}^{+} \leq 1$.

This new fuzzy set is the extension of the IFS and differs from IFS by including a circle of the number consisting of membership and non-membership degrees (Kahraman & Alkan, 2021).

Definition 3. Atanassov, 2020; Atanassov & Marinov, 2021 *Let E be a fixed universe, and a generic element a C-IFS* C_r *in E is denoted by x;*

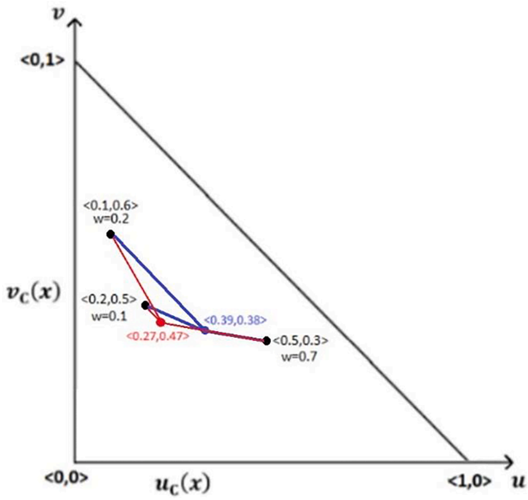

Fig. 5. C-IFS C_i results of sample set.

C_r={<x : $u_C(x)$,$v_C(x)$;r>, $x \in E$} is the form of an object that is the C-IFS, where the functions "u, v: E→[0, 1]" define respectively the membership function and the non-membership function of the element $x \in E$ to the set C-IFS with the condition:

$$0 \leq \ u_C(x) + v_C(x) \ \leq 1 \text{ and } r \in [0, \sqrt{2}] \tag{3}$$

where r is the radius of the circle around each element $x \in E$. The indeterminacy function can be also defined as $\pi_C(x) = 1 - u_C(x) - v_C(x)$. The geometrical presentation of C-IFS is in Fig. 3.

When $r = 0$, a C-IFS is reduced to a standard IFS. As the Atanassov discussed in the article (Atanassov & Marinov, 2021), the region of the r values should be $[0, \sqrt{2}]$ to allow the point $< 0{,}1 >$ and $< 1{,}0 >$ cover IFIT. Triangle coverage of different r values is illustrated in Fig. 4.

Definition 4. Atanassov, 2020 *Let $\{< m_{i,1}, n_{i,1} >, < m_{i,2}, n_{i,2} >, \cdots\}$ is a set of IF pairs. The C-IFS C_i is calculated from* pairs *where i is the number of the IFS and k_i is the number of IF pairs in each set. The arithmetic average of* the set *is as follows*:

$$C_i = < u_C(C_i), v_C(C_i) > = < \frac{\sum_{j=1}^{k_i} m_{i,j}}{k_i}, \frac{\sum_{j=1}^{k_i} n_{i,j}}{k_i} > \tag{4}$$

Here, we propose to use the weighted arithmetic average means of the set to also consider the weights of each element. Let $\{< m_{i,1}, n_{i,1} >, < m_{i,2}, n_{i,2} >, \cdots\}$ is a set of IF pairs and $W_i = \{w_{i,1}, \cdots, w_{i,k_i}\}$ is the set of the weights of IF pairs where $w_{i,j} \in [0, 1]$ and $\sum_{j=1}^{k_i} w_{i,j} = 1$, and k_i is the number of IF pairs in each set. The C-IFS C_i is calculated from pairs where i is the number of the IFS. The weighted arithmetic average of the set is as follows:

$$C_i = < u_C(C_i), v_C(C_i) > = < \sum_{j=1}^{k_i} w_{i,j} m_{i,j}, \sum_{j=1}^{k_i} w_{i,j} n_{i,j} > \tag{5}$$

The difference between the C-IFS C_i is found by the weighted average and the arithmetic mean is illustrated in Fig. 5 for a sample set of IF pairs as $\{< 0.2, 0.5 >, < 0.5, 0.3 >, < 0.1, 0.6 >\}$ with weight $W_i = \{0.1, 0.7, 0.2\}$. The C-IFS C_i is calculated as $< 0.27{,}0.47 >$ by Eq. (4) and $< 0.39{,}0.38 >$ by Eq. (5).

One step forward, C_i can be calculated by aggregation operators. According to the case, various operators should be selected. Two existing arithmetic operators on IFS are given to aggregate IF pairs as follows:

Definition 5. Zeshui, 2007 *Let $\{< m_{i,1}, n_{i,1} >, < m_{i,2}, n_{i,2} >, \cdots\}$ is a set of IF pairs. Then, their aggregated value which is the center of C_i by using the IF weighted averaging (IFWA) operator is also an IF value*:

$$C_i = IFWA_{W_i}(\langle m_{i,1}, n_{i,1} >, < m_{i,2}, n_{i,2} >, \cdots) = \\ < 1 - \prod_{j=1}^{n} (1 - m_{i,j})^{w_{i,j}}, \prod_{j=1}^{n} n_{i,j}^{\ w_{i,j}} > \tag{6}$$

where $W_i = \{w_{i,1}, \cdots, w_{i,n}\}$ is the weighting vector of IF pairs with $w_{i,j} \in [0, 1]$ and $\sum_{j=1}^{n} w_{i,j} = 1$.

Definition 6. Zeshui, 2007 *Let $\{< m_{i,1}, n_{i,1} >, < m_{i,2}, n_{i,2} >, \cdots\}$ is a set of IF pairs. Then, their aggregated value which is the center of C_i by using the IF weighted geometric (IFGA) operator is also an IF value*:

$$C_i = IFWG_{W_i}(< m_{i,1}, n_{i,1} >, < m_{i,2}, n_{i,2} >, \cdots) = \\ < \prod_{j=1}^{n} m_{i,j}^{\ w_{i,j}}, 1 - \prod_{j=1}^{n} (1 - n_{i,j})^{w_{i,j}} > \tag{7}$$

where $W_i = \{w_{i,1}, \cdots, w_{i,n}\}$ is the weighting vector of IF pairs with $w_{i,j} \in [0, 1]$ and $\sum_{j=1}^{n} w_{i,j} = 1$.

The radius r_i of the C_i is obtained by the maximum of the Euclidean distances as follows (Atanassov, 2020):

$$r_i = \max_{1 \leq j \leq k_i} \sqrt{\left(u_C(C_i) - m_{i,j}\right)^2 + \left(v_C(C_i) - n_{i,j}\right)^2} \tag{8}$$

Definition 7. Atanassov, 2020 *Let $L^* = \{< a, b > | a, b \in [0, 1] \& a + b \leq 1\}$. Therefore C_r can be rewritten in the form $C_r^{\ *}$={<x : $O_r(u_C(x), v_C(x))$)>, $x \in E$} where "$O_r(u_C(x), v_C(x)) = \{< a, b > \Big| a, b \in [0, 1], \sqrt{(u_C(x) - a)^2 + (v_C(x) - b)^2} \leq r, a + b \leq 1\}$" is a function of circle representation.*

Definition 8. Atanassov, 2020 *Let $C_1 = < u_{C_1}(x), v_{C_1}(x); r_1 >$ and $C_2 = < u_{C_2}(x), v_{C_2}(x); r_2 >$ be two C-IFSs. The basic operations are given by taking into account the maximum and minimum of the radiuses as follows*:

$$C_1 \oplus_{min} C_2 = \{ \\ < x, u_{C_1}(x) + u_{C_2}(x) - u_{C_1}(x)^* u_{C_2}(x), v_{C_1}(x)^* v_{C_2}(x);\ min(r_1, r_2) \\ > | \ x \in E\} \tag{9}$$

$$C_1 \oplus_{max} C_2 = \{ \\ < x, u_{C_1}(x) + u_{C_2}(x) - u_{C_1}(x)^* u_{C_2}(x), v_{C_1}(x)^* v_{C_2}(x);\ max(r_1, r_2) \\ > | \ x \in E\} \tag{10}$$

$$C_1 \otimes_{min} C_2 = \{< x, u_{C_1}(x)^* u_{C_2}(x), v_{C_1}(x) + v_{C_2}(x) - v_{C_1}(x)^* v_{C_2}(x);\ min(r_1, r_2) \\ > | \ x \in E\} \tag{11}$$

$$C_1 \otimes_{max} C_2 = \{ \\ < x, u_{C_1}(x)^* u_{C_2}(x), v_{C_1}(x) + v_{C_2}(x) - v_{C_1}(x)^* v_{C_2}(x);\ max(r_1, r_2) \\ > | \ x \in E\} \tag{12}$$

The complement of the C-IFS number $C_1 = < u_{C_1}(x), v_{C_1}(x); r_1 >$ is defined as follows (Atanassov, 2020):

$$C_1^{\ C} = < v_{C_1}(x), u_{C_1}(x); r_1 > \tag{13}$$

To enlarge its usage in methods, C-IFS needs the defuzzification function. As in IFS and IVIFS, the score and accuracy function should be defined. Here, we propose novel score and accuracy functions for C-IFS based on different perspectives. First, to clarify where the functions are derived from, the defuzzification formulas for IFS and IVIFS are given below:

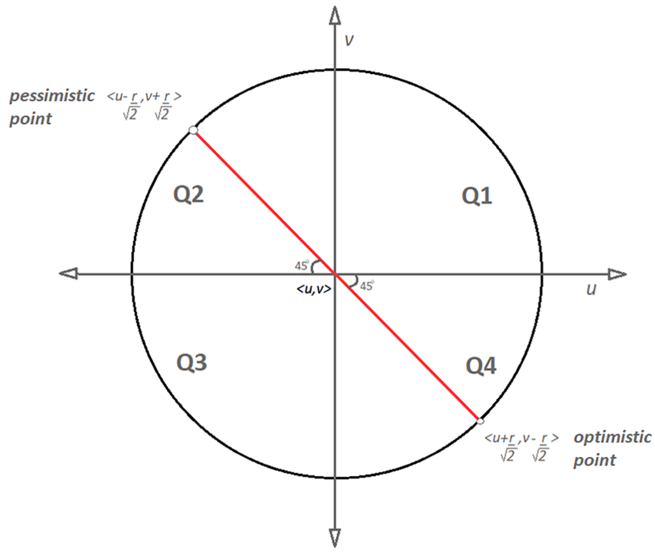

Fig. 6. Optimistic and pessimistic points of C-IFS.

Definition 9. Chen & Tan, 1994; Hong & Choi, 2000 *Let $\widetilde{a} = (u_{\widetilde{a}}, v_{\widetilde{a}})$ be an IFS, a score function S_{IFS} and an accuracy function H_{IFS} of the IFV $\widetilde{a}$ are defined as follows*:

$$S_{IFS}(\widetilde{a}) = u_{\widetilde{a}} - v_{\widetilde{a}} \text{where} S_{IFS}(\widetilde{a}) \in [-1, 1] \tag{14}$$

$$H_{IFS}(\widetilde{a}) = u_{\widetilde{a}} + v_{\widetilde{a}} \text{where} H_{IFS}(\widetilde{a}) \in [0, 1] \tag{15}$$

Definition 10. Ze-Shui, 2007 *Let $\widetilde{A} = \left([u_{\widetilde{A}}^{-}, u_{\widetilde{A}}^{+}], [v_{\widetilde{A}}^{-}, v_{\widetilde{A}}^{+}]\right)$ be an IVIFS, the score function S_{IVIFS} and the accuracy function H_{IVIFS} of the IFV $\widetilde{a}$ are defined as follows*:

$$S_{IVIFS}\left(\widetilde{A}\right) = \frac{S_{IFS}\left(\langle u_{\widetilde{A}}^{-}, v_{\widetilde{A}}^{-}\right) + S_{IFS}\left(\langle u_{\widetilde{A}}^{+}, v_{\widetilde{A}}^{+}\right)}{2} = \frac{u_{\widetilde{A}}^{-} + u_{\widetilde{A}}^{+} - v_{\widetilde{A}}^{-} - v_{\widetilde{A}}^{+}}{2}, \text{ where } S_{IVIFS}\left(\widetilde{A}\right) \in [-1, 1] \tag{16}$$

$$H_{IVIFS}\left(\widetilde{A}\right) = \frac{H_{IFS}\left(\langle u_{\widetilde{A}}^{-}, v_{\widetilde{A}}^{-}\right) + H_{IFS}\left(\langle u_{\widetilde{A}}^{+}, v_{\widetilde{A}}^{+}\right)}{2} = \frac{u_{\widetilde{A}}^{-} + u_{\widetilde{A}}^{+} + v_{\widetilde{A}}^{-} + v_{\widetilde{A}}^{+}}{2}, \text{ where } H_{IVIFS}\left(\widetilde{A}\right) \in [0, 1] \tag{17}$$

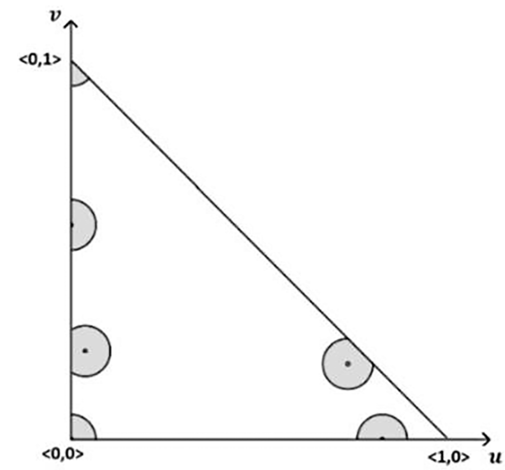

Fig. 7. The C-IFS whose entire circle is not in IFIT.

Therefore, it is possible to generate a score value using the points included in the C-IFS.

To interpret the values in C-IFS, divide it into four equal parts. According to Fig. 6, the values in the first quarter (Q1) can be reached by increasing the u and v values from the central IFS *(u,v)*. This means increasing membership and non-membership values. The values in the second quartile (Q2) can be obtained from IFS with the center *(u,v)* by decreasing the value of u and increasing the value of v. Here, when the angle is taken to 45^0 and the point is r distance away, the lowest membership value and the highest non-membership value of C-IFS are found. This point is called the “pessimistic point” of the C-IFS, and this point is also an IFS. Conversely, going to the point r away by 45^0 angle, in the fourth quarter (Q4), the C-IFS has the point with the highest membership and lowest non-member values. This point is called the “optimistic point” of the C-IFS, and this point is also an IFS. Hence, similar to the idea of using endpoints in IVIFS, the score and accuracy values of a C-IFS can be found using two points determined in the circle.

Definition 11. *Here, a new score (S_{C-IFS}) and an accuracy (H_{C-IFS}) function for C-IFS are defined. Let $c = (u_c, v_c; r)$ be an C-IFV with the optimistic point is $< u_c + \frac{r}{\sqrt{2}}, v_c - \frac{r}{\sqrt{2}} >$ and the pessimistic point is $< u_c - \frac{r}{\sqrt{2}}, v_c + \frac{r}{\sqrt{2}} >$. A score function S_{C-IFS} and an accuracy function H_{C-IFS} of the C-IFV c are defined as follows with respect to the decision-maker's (or manager's) preference information $\lambda \in [0, 1]$*:

$$S_{C-IFS}(c) = \frac{\lambda^* S_{IFS}\left(\left\langle u_c + \frac{r}{\sqrt{2}}, v_c - \frac{r}{\sqrt{2}} >\right) + (1-\lambda)^* S_{IFS}\left(\left\langle u_c - \frac{r}{\sqrt{2}}, v_c + \frac{r}{\sqrt{2}} >\right)}{3} = \frac{u_c - v_c + \sqrt{2}r(2\lambda - 1)}{3}, \text{ where } S_{C-IFS}(c) \in [-1, 1] \tag{18}$$

$$H_{C-IFS}(c) = \lambda^* H_{IFS}\left(\left\langle u_c + \frac{r}{\sqrt{2}}, v_c - \frac{r}{\sqrt{2}} >\right) + (1-\lambda)^* H_{IFS}\left(\left\langle u_c - \frac{r}{\sqrt{2}}, v_c + \frac{r}{\sqrt{2}} >\right) = u_c + v_c, \text{ where } H_{C-IFS}(c) \in [0, 1] \tag{19}$$

Before suggesting defuzzification functions, let's explain how to use them in C-IFS. C-IFS basically covers a circle with a center IFS point and radius r around that point. Each point in C-IFS is actually an IFS. λ reflects the decision-maker's perspective to the model. If λ is equal to zero, it shows the full pessimistic point of view, and if λ is equal to one, it shows the optimistic point of view. In general acceptance, $\lambda \in [0, 0.5)$

Table 2
Intuitionistic fuzzy linguistic terms.

Linguistic Terms	Code	IFN
Absolutely Low	AL	<0.1,0.9>
Very Low	VL	<0.2,0.75>
Low	L	<0.3,0.65>
Medium Low	ML	<0.4,0.55>
Fair	F	<0.5,0.45>
Medium High	MH	<0.6,0.35>
High	H	<0.7,0.25>
Very High	VH	<0.8,0.15>
Absolutely High	AH	<0.9,0.1>

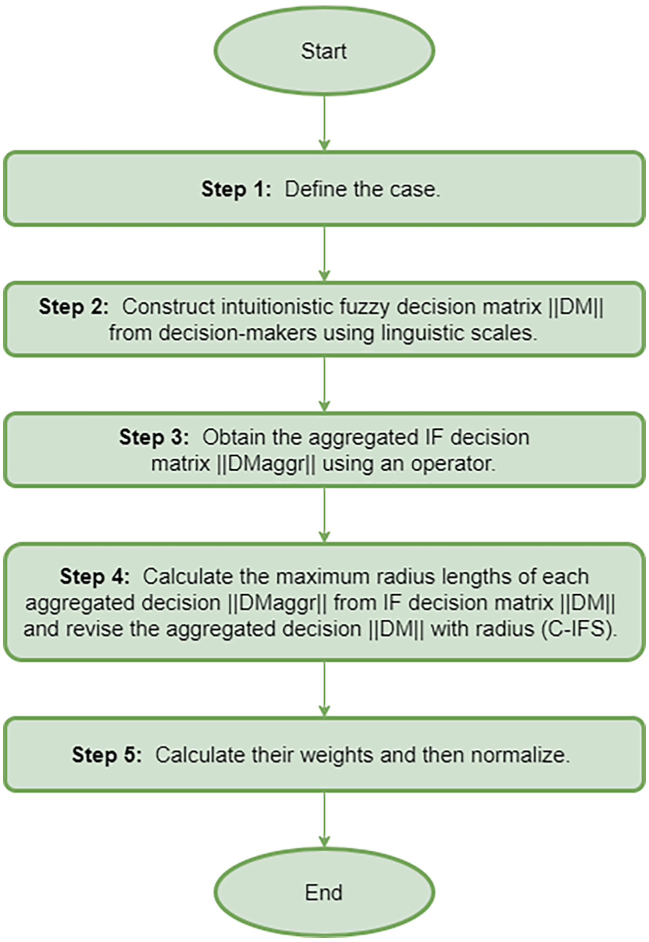

Fig. 8. Flowchart of deriving criteria weights from C-IFS.

indicates a pessimistic point of view, and $\lambda \in (0.5, 1]$ indicates an optimistic point of view. $\lambda = 0.5$ reflects the indifferent attitude of the decision-maker.

Although these functions work by definition, the general formulation covering all situations should also be revised. From Fig. 7, a C-IFS circle may not be entirely within IFIT. In this case, the above formulas will not be sufficient to determine the optimistic and pessimistic points.

Therefore, the pessimistic point should be determined as $< u_c - min\left\{\frac{r}{\sqrt{2}}, u_c\right\}, \min\left\{1, v_c + max\left\{\frac{r}{\sqrt{2}}, \sqrt{r^2 - u_c^2}\right\}\right\}\rangle$ and the optimistic point should be determined as $< \min\left\{1, u_c + max\left\{\frac{r}{\sqrt{2}}, \sqrt{r^2 - v_c^2}\right\}\right\}, v_c - min\left\{\frac{r}{\sqrt{2}}, v_c\right\}\rangle$. Let's realize that, theoretically, a C-IFS of radius r is possible with the center (0,0), (1.0) or (0.1) which are IFIT vertices. However, it is not possible for these centers to be formed in C-IFSs created by combining IFS numbers. Because there is no IF point outside IFIT, it is not possible to calculate these vertices as centers.

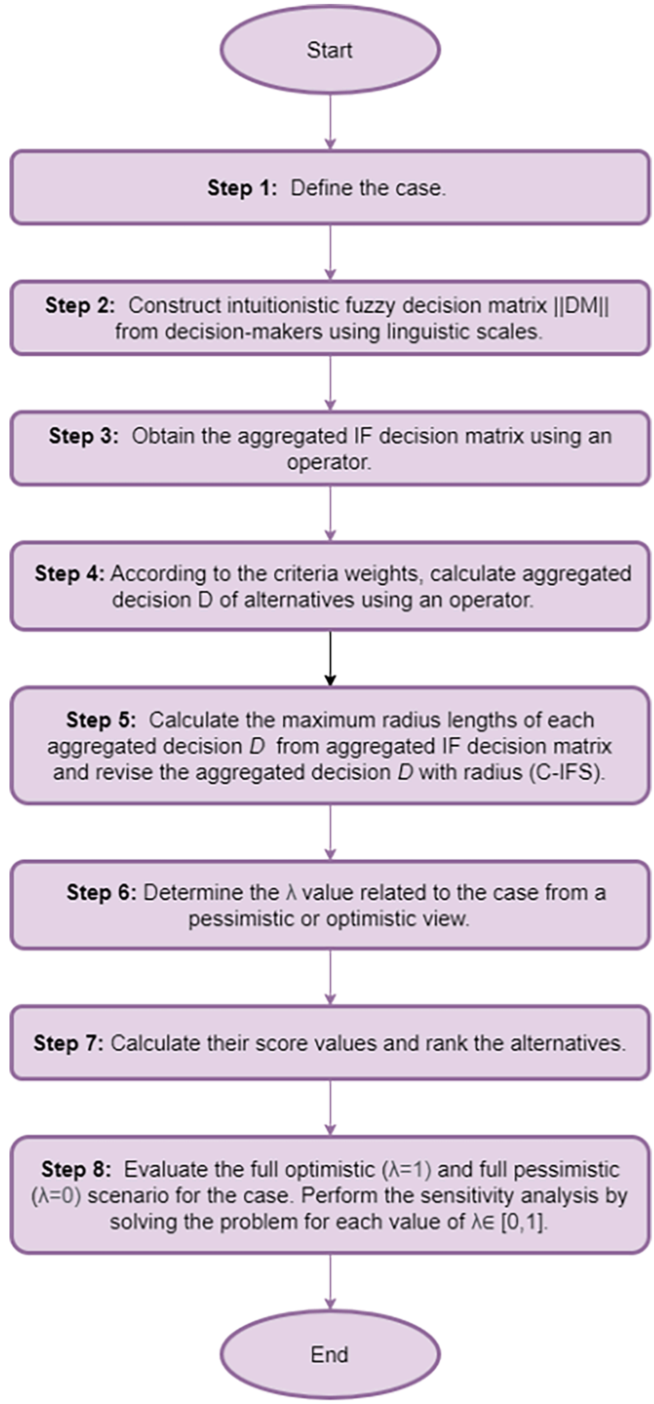

Fig. 9. Procedure of multi criteria decision making based on C-IFS.

Definition 12. *Let* $c_1 =< u_{c_1}, v_{c_1}; r >$ *and* $c_2 =< u_{c_2}, v_{c_2}; r >$ *be two C-IFSs. Then, the ranking rule is defined as follows*:

- If $S_c(c_1) \succ S_c(c_2)$, then $c_1 \succ c_2$.
- If $S_c(c_1) = S_c(c_2)$, then.
- If $H_c(c_1) \succ H_c(c_2)$, then $c_1 \succ c_2$.
- If $H_c(c_1) = H_c(c_2)$, then $c_1 = c_2$.

Table 3
The aggregated IF decision matrix by Eqs. (4)-(7).

Eq. (4):	$< \frac{\sum_{j=1}^{k_i} m_{ij}}{k_i}, \frac{\sum_{j=1}^{k_i} n_{ij}}{k_i} >$	$\|\|DM_{aggr}\|\| = \begin{matrix} C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \end{matrix} \begin{bmatrix} < 0.34, 0.62 > \\ < 0.38, 0.57 > \\ < 0.32, 0.63 > \\ < 0.66, 0.29 > \\ < 0.48, 0.47 > \end{bmatrix}$
Eq. (5):	$< \sum_{j=1}^{k_i} w_{i,j} m_{ij}, \sum_{j=1}^{k_i} w_{i,j} n_{ij} >$	$\|\|DM_{aggr}\|\| = \begin{matrix} C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \end{matrix} \begin{bmatrix} < 0.33, 0.63 > \\ < 0.37, 0.58 > \\ < 0.34, 0.62 > \\ < 0.68, 0.28 > \\ < 0.50, 0.46 > \end{bmatrix}$
Eq. (6):	$< 1 - \prod_{j=1}^{n} (1 - m_{ij})^{w_{ij}}, \prod_{j=1}^{n} n_{i,j}^{w_{ij}} >$	$\|\|DM_{aggr}\|\| = \begin{matrix} C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \end{matrix} \begin{bmatrix} < 0.34, 0.28 > \\ < 0.39, 0.34 > \\ < 0.35, 0.32 > \\ < 0.69, 0.67 > \\ < 0.51, 0.48 > \end{bmatrix}$
Eq. (7):	$< \prod_{j=1}^{n} m_{ij}^{w_{ij}}, 1 - \prod_{j=1}^{n} (1 - n_{ij})^{w_{ij}} >$	$\|\|DM_{aggr}\|\| = \begin{matrix} C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \end{matrix} \begin{bmatrix} < 0.28, 0.68 > \\ < 0.34, 0.61 > \\ < 0.32, 0.63 > \\ < 0.67, 0.28 > \\ < 0.48, 0.47 > \end{bmatrix}$

3. Decision making using C-IFS

This section introduces the new circular intuitionistic fuzzy MCDM method step by step with novel functions. The methodology aims to determine the rank order of the alternatives according to the criteria and the reviews of decision-makers. The steps of the proposed C-IFS MCDM methodology are explained as follows.

3.1. Deriving criteria weights

Step 1: Define the case. Consider "$C = \{C_1, C_2, \cdots, C_n\}$" is the criteria set and "$D = \{D_1, D_2, \cdots, D_k\}$" is the set of decision-makers. Let "$W_D = \{W_{D_1}, W_{D_2}, \cdots, W_{D_k}\}$" is the weight vector of criteria where $W_{D_i} \geq 0$ and $\Sigma W_{D_i} = 1$ (The cost type criteria should be converted benefit type by Eq. (2)).

Step 2: Construct intuitionistic fuzzy decision matrix $||DM||$ from decision-makers using linguistic scales in Table 2.

Step 3: Obtain the aggregated IF decision matrix $||DM_{aggr}||$ using an operator by Eqs. (4)-(7). (For Eqs. (5)–(7), when the weights of the decision-makers are considered equal, $W_{D_i} = 1/k$).

Step 4: Calculate the maximum radius lengths of each aggregated decision $||DM_{aggr}||$ by Eq. (8) from IF decision matrix $||DM||$ and revise the aggregated decision $||DM||$ with radius (C-IFS).

Step 5: Calculate their weights by $W_{C_i} = \frac{u_c - v_c + \sqrt{2}r + 1}{4}$ where $W_{C_i} \in [0, 1]$ and then normalize.

The flowchart of deriving criteria weights from C-IFS is represented in Fig. 8.

3.2. Multi criteria decision making

Step 1: Define the case. Consider "$A = \{A_1, A_2, \cdots, A_m\}$" is the alternative set, "$C = \{C_1, C_2, \cdots, C_n\}$" is the criteria set and "$D = \{D_1, D_2, \cdots, D_k\}$" is the set of decision-makers. Let "$W_C = \{W_{C_1}, W_{C_2}, \cdots, W_{C_n}\}$" is the weight vector of criteria where $W_{C_i} \geq 0$ and $\Sigma W_{C_i} = 1$. These weight vectors are determined by decision-makers or calculated by the algorithm given in Section 3.1.

Step 2: Collect intuitionistic fuzzy decision matrices from decision-makers using linguistic scales in Table 2.

Step 3: Obtain the aggregated IF decision matrix using an operator by Eqs. (6)-(7).

Step 4: According to the criteria weights, calculate aggregated decision $\widetilde{D}$ of alternatives using an operator by Eqs. (6)-(7).

Table 4
Criteria weights.

Criterion	Normalized Score
C_1	0.180
C_2	0.189
C_3	0.159
C_4	0.265
C_5	0.217

Step 5: Calculate the maximum radius lengths of each aggregated decision $\widetilde{D}$ by Eq. (8) from aggregated IF decision matrix and revise the aggregated decision $\widetilde{D}$ with radius (C-IFS).

Step 6: Determine the λ value related to the case from a pessimistic or optimistic view.

Step 7: Calculate their score values by Eq. (18) and rank the alternatives.

Step 8: Evaluate the full optimistic ($\lambda = 1$) and full pessimistic ($\lambda = 0$) scenario for the case. Perform the sensitivity analysis by solving the problem for each value of $\lambda \in [0, 1]$.

The procedure of multi criteria decision making based on C-IFS is illustrated in Fig. 9.

4. Illustrative example

It is crucial to cooperate with appropriate suppliers to establish a seamless supply chain network. The performance of the selected supplier is directly related to the success of the supply chain management. Therefore, a supplier that can contribute to the goals of the organization should be selected.

As a case study, the evaluation of alternative suppliers for a manufacturing company in Turkey is discussed. This company is looking for the best supplier to work with on the main raw material supply. Decision-makers and literature review are based on identifying the criteria taken into consideration in the process of making the selection decision. In order to rank alternative suppliers, initially, the weights of the criteria should be calculated.

4.1. Deriving criteria weights

Step 1: With the assessments of the interviewed decision-makers, a wide list of criteria is created, and five criteria that are considered suitable for the requirements of the sector and the objectives of the company are determined. The criteria set is "$C = \{C_1, C_2, C_3, C_4, C_5\}$" with C_1 : delivery performance, C_2 : agility, C_3 : technology, C_4 : quality, C_5 : collaboration level, and "$DM = \{DM_1, DM_2, DM_3, DM_4, DM_5\}$" is the set of decision-makers. All criteria are benefit types. The weight vector of criteria is "$W_D = \{W_{D_1}, W_{D_2}, W_{D_3}, W_{D_4}, W_{D_5}\}$" with $W_D = \{0.2, 0.3, 0.25, 0.1, 0.15\}$.

Step 2: The intuitionistic fuzzy decision matrix $||DM||$ is constructed as follows by from decision-makers using linguistic scales in Table 2.

$$||DM||_{5x5} = \begin{matrix} & DM_1 & DM_2 & DM_3 & DM_4 & DM_5 \\ C_1 & F & VL & MH & F & H \\ C_2 & ML & F & VL & MH & VL \\ C_3 & VL & L & F & MH & ML \\ C_4 & MH & H & ML & H & F \\ C_5 & L & MH & F & L & MH \end{matrix}$$

Step 3: The aggregated IF decision matrix $||DM_{aggr}||$ is obtained using the operators by Eqs. (4)-(7) as in Table 3.

Table 5
Intuitionistic fuzzy decision matrices from decision-makers.

	DM1					DM2					DM3					DM4					DM5				
	C_1	C_2	C_3	C_4	C_5	C_1	C_2	C_3	C_4	C_5	C_1	C_2	C_3	C_4	C_5	C_1	C_2	C_3	C_4	C_5	C_1	C_2	C_3	C_4	C_5
A_1	F	MH	AL	ML	H	MH	ML	MH	H	ML	VH	H	L	MH	ML	ML	MH	VL	ML	H	MH	ML	H	VH	F
A_2	ML	MH	H	ML	VH	ML	F	VL	MH	F	F	MH	MH	ML	VL	L	F	VL	L	ML	H	F	MH	H	MH
A_3	L	MH	H	VH	F	L	H	VL	MH	L	MH	F	H	L	ML	F	VL	L	VL	ML	MH	ML	H	VH	MH
A_4	MH	ML	AL	H	L	H	ML	F	H	VL	F	ML	H	MH	F	VL	MH	F	VL	H	F	MH	H	F	VH
A_5	VL	ML	L	VH	MH	F	MH	VL	F	VL	ML	F	VL	H	L	F	MH	ML	H	L	F	MH	ML	VH	ML
A_6	F	F	H	MH	ML	MH	L	F	ML	ML	ML	F	VL	L	F	ML	MH	ML	VL	H	L	ML	MH	H	VH
A_7	VH	VL	ML	L	VL	VH	VL	ML	L	ML	L	F	ML	VL	H	MH	F	VL	MH	AL	VH	AH	L	MH	MH

Step 4: This case proceeds with the aggregated IF by Eq. (5). The maximum radius lengths of each aggregated decision $||DM||_{5x1}$ is calculated by Eq. (8) from IF decision matrix $||DM||$. The aggregated decision $||DM||_{5x1}$ with radius (C-IFS) is revised as follows:

$$||DM_{aggr}|| = \begin{matrix} C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \end{matrix} \begin{bmatrix} < 0.33, 0.63; 0.35 > \\ < 0.37, 0.58; 0.33 > \\ < 0.34, 0.62; 0.23 > \\ < 0.68, 0.28; 0.25 > \\ < 0.50, 0.46; 0.28 > \end{bmatrix}$$

Step 5: The criteria weights are calculated by $W_{C_i} = \frac{u_c - v_c + \sqrt{2}r + 1}{4}$ where $W_{C_i} \in [0, 1]$ and then normalized as in Table 4.

4.2. Multi criteria decision making (ranking alternatives)

Step 1: The weights of the selection criteria of the company in the case study were calculated above section. Seven national alternative suppliers are determined to meet the raw material needs of the company. The alternative set is "$A = \{A_1, A_2, A_3, A_4, A_5, A_6, A_7\}$". Five decision-makers from Section 4.1 are asked to evaluate alternative suppliers according to the criteria with calculated weights in Table 4.

Step 2: The intuitionistic fuzzy decision matrices from decision-makers are collected as in Table 5 using linguistic scales in Table 2.

Step 3: The aggregated IF decision matrix is obtained using the operators by Eqs. (6)-(7) as follows.

$$DM_{IFWA} = \begin{matrix} & C_1 & C_2 & C_3 & C_4 & C_5 \\ A_1 & < 0.63,0.31 > & < 0.55,0.39 > & < 0.44,0.51 > & < 0.63,0.32 > & < 0.53,0.42 > \\ A_2 & < 0.48,0.47 > & < 0.56,0.39 > & < 0.50,0.44 > & < 0.51,0.43 > & < 0.54,0.40 > \\ A_3 & < 0.46,0.49 > & < 0.56,0.39 > & < 0.56,0.38 > & < 0.61,0.33 > & < 0.43,0.52 > \\ A_4 & < 0.57,0.38 > & < 0.46,0.49 > & < 0.54,0.41 > & < 0.62,0.33 > & < 0.49,0.45 > \\ A_5 & < 0.43,0.52 > & < 0.54,0.41 > & < 0.28,0.67 > & < 0.70,0.25 > & < 0.36,0.58 > \\ A_6 & < 0.48,0.47 > & < 0.44,0.50 > & < 0.50,0.45 > & < 0.47,0.48 > & < 0.55,0.40 > \\ A_7 & < 0.71,0.24 > & < 0.50,0.46 > & < 0.37,0.58 > & < 0.37,0.58 > & < 0.48,0.47 > \end{matrix}$$

$$DM_{IFWG} = \begin{matrix} & C_1 & C_2 & C_3 & C_4 & C_5 \\ A_1 & < 0.60,0.35 > & < 0.52,0.43 > & < 0.32,0.64 > & < 0.58,0.37 > & < 0.49,0.46 > \\ A_2 & < 0.45,0.50 > & < 0.55,0.40 > & < 0.40,0.54 > & < 0.48,0.47 > & < 0.44,0.50 > \\ A_3 & < 0.42,0.53 > & < 0.51,0.44 > & < 0.44,0.50 > & < 0.50,0.44 > & < 0.41,0.54 > \\ A_4 & < 0.52,0.42 > & < 0.44,0.51 > & < 0.41,0.56 > & < 0.56,0.38 > & < 0.38,0.56 > \\ A_5 & < 0.39,0.55 > & < 0.53,0.42 > & < 0.26,0.69 > & < 0.66,0.29 > & < 0.32,0.63 > \\ A_6 & < 0.45,0.50 > & < 0.42,0.53 > & < 0.43,0.52 > & < 0.41,0.54 > & < 0.50,0.45 > \\ A_7 & < 0.61,0.34 > & < 0.35,0.60 > & < 0.36,0.59 > & < 0.32,0.62 > & < 0.37,0.59 > \end{matrix}$$

Step 4: According to the criteria weights, the aggregated decision $\widetilde{D}$ of alternatives is calculated using the operators by Eqs. (6)-(7) as follows.

$$\widetilde{D}_{IFWA} = \begin{matrix} A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \\ A_6 \\ A_7 \end{matrix} \begin{bmatrix} <0.57, 0.56> \\ <0.52, 0.51> \\ <0.54, 0.52> \\ <0.55, 0.53> \\ <0.51, 0.45> \\ <0.49, 0.48> \\ <0.50, 0.46> \end{bmatrix}$$

$$\widetilde{D}_{IFWG} = \begin{matrix} A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \\ A_6 \\ A_7 \end{matrix} \begin{bmatrix} <0.50, 0.45> \\ <0.46, 0.49> \\ <0.45, 0.49> \\ <0.46, 0.49> \\ <0.42, 0.52> \\ <0.44, 0.51> \\ <0.38, 0.57> \end{bmatrix}$$

Step 5: The maximum radius lengths of each aggregated decision $\widetilde{D}$ are calculated by Eq. (8) from aggregated IF decision matrix. The aggregated decision $\widetilde{D}$ with radius (C-IFS) is revised.

$$\widetilde{D}_{IFWA} = \begin{matrix} A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \\ A_6 \\ A_7 \end{matrix} \begin{bmatrix} <0.57, 0.56; 0.25> \\ <0.52, 0.51; 0.13> \\ <0.54, 0.52; 0.21> \\ <0.55, 0.53; 0.21> \\ <0.51, 0.45; 0.32> \\ <0.49, 0.48; 0.10> \\ <0.50, 0.46; 0.31> \end{bmatrix}$$

$$\widetilde{D}_{IFWG} = \begin{matrix} A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \\ A_6 \\ A_7 \end{matrix} \begin{bmatrix} <0.50, 0.45; 0.26> \\ <0.46, 0.49; 0.13> \\ <0.45, 0.49; 0.08> \\ <0.46, 0.49; 0.15> \\ <0.42, 0.52; 0.34> \\ <0.44, 0.51; 0.08> \\ <0.38, 0.57; 0.33> \end{bmatrix}$$

Step 6: The λ value is determined as 0.70 by decision-makers which reflects their optimistic view on case.

Step 7: The score values are obtained by Eq. as in Table 6. The ranking of the alternatives is $A_5 \succ A_7 \succ A_1 \succ A_4 \succ A_3 \succ A_2 \succ A_6$ from $\widetilde{D}_{IFWA}$ and $A_1 \succ A_5 \succ A_4 \succ A_2 \succ A_3 \succ A_7 \succ A_6$ from $\widetilde{D}_{IFWG}$.

Step 8: For each value of $\lambda \in [0, 1]$, the values and ranking results are given in Table 7 and Table 8. Fig. 10 and Fig. 11 illustrate the changes in the rankings.

Table 6
Scores of alternatives.

Alternative	Score of $\widetilde{D}_{IFWA}$	Score of $\widetilde{D}_{IFWG}$
A_1	0.053	0.064
A_2	0.027	0.015
A_3	0.045	0.001
A_4	0.045	0.019
A_5	0.079	0.030
A_6	0.022	−0.009
A_7	0.070	−0.002

4.3. Comparing C-IFS MCDM with IF MCDM

Based on the proposed methodology, C-IFS MCDM can actually be considered as an extension of IF-MCDM procedures. Therefore, it would be appropriate to compare the result of the proposed case with both methods in order to prove the validity of C-IFS MCDM.

Let's re-examine the supplier selection for seamless supply chain network problem discussed in Section 4 in terms of IF numbers. IF matrices given by the decision-makers will be used without changing. When the steps in sub-section 4.1 are processed to derive the weights of the criteria;

- **Step 1:** Same as Step 1 in Section 4.1.
- **Step 2:** Same as Step 2 in Section 4.1.
- **Step 3:** Since the radius is not used in IF numbers, we can proceed over the results obtained from the aggregation functions Eq. (6) and Eq. (7). Here, the geometric weighted average operator was used for comparison and the aggregation matrix resulting from the Eq. (7) in Table 3 as follows.

$$||DM_{aggr}|| = \begin{matrix} C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \end{matrix} \begin{bmatrix} <0.28, 0.68> \\ <0.34, 0.61> \\ <0.32, 0.63> \\ <0.67, 0.28> \\ <0.48, 0.47> \end{bmatrix}$$

- **Step 4:** Since radius is not used in IF numbers, this step does not apply.
- **Step 5:** Eq. (14) and Eq. (15) should be used to defuzzify the aggregated IF matrices. The normalized results are given in Table 9.

When the steps in sub-section 4.2 are processed to rank the alternatives;

- **Step 1:** Same as Step 1 in Section 4.2.
- **Step 2:** Same as Step 2 in Section 4.2.
- **Step 3:** Same as Step 3 in Section 4.2.
- **Step 4:** According to the criteria weights calculated in Table 9, the aggregated decision $\widetilde{D}$ of alternatives is calculated using the operators by Eqs. (6)-(7) as follows.

$$\widetilde{D}_{IFWA} = \begin{matrix} A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \\ A_6 \\ A_7 \end{matrix} \begin{bmatrix} <0.572, 0.556> \\ <0.523, 0.515> \\ <0.537, 0.520> \\ <0.547, 0.533> \\ <0.509, 0.454> \\ <0.491, 0.482> \\ <0.498, 0.461> \end{bmatrix}$$

$$\widetilde{D}_{IFWG} = \begin{matrix} A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \\ A_6 \\ A_7 \end{matrix} \begin{bmatrix} <0.499, 0.453> \\ <0.459, 0.487> \\ <0.452, 0.494> \\ <0.462, 0.488> \\ <0.423, 0.522> \\ <0.436, 0.511> \\ <0.379, 0.569> \end{bmatrix}$$

- **Step 5:** Since radius is not used in IF numbers, this step does not apply.
- **Step 6:** Since λ value is not used in IF numbers, this step does not apply.

Table 7
Sensitivity analysis for λ value for $\widetilde{D}_{IFWA}$.

Score of $\widetilde{D}_{IFWA}$	A_1	A_2	A_3	A_4	A_5	A_6	A_7	Ranking
$\lambda = 0$	−0.114	−0.057	−0.092	−0.096	−0.133	−0.044	−0.132	$A_6 \succ A_2 \succ A_3 \succ A_4 \succ A_1 \succ A_7 \succ A_5$
$\lambda = 0.1$	−0.090	−0.045	−0.072	−0.076	−0.103	−0.035	−0.103	$A_6 \succ A_2 \succ A_3 \succ A_4 \succ A_1 \succ A_5 \succ A_7$
$\lambda = 0.2$	−0.066	−0.033	−0.053	−0.056	−0.072	−0.025	−0.074	$A_6 \succ A_2 \succ A_3 \succ A_4 \succ A_1 \succ A_5 \succ A_7$
$\lambda = 0.3$	−0.042	−0.021	−0.033	−0.036	−0.042	−0.016	−0.045	$A_6 \succ A_2 \succ A_3 \succ A_4 \succ A_5 \succ A_1 \succ A_7$
$\lambda = 0.4$	−0.019	−0.009	−0.014	−0.015	−0.012	−0.006	−0.017	$A_6 \succ A_2 \succ A_5 \succ A_3 \succ A_4 \succ A_7 \succ A_1$
$\lambda = 0.5$	0.005	0.003	0.006	0.005	0.019	0.003	0.012	$A_5 \succ A_7 \succ A_3 \succ A_1 \succ A_4 \succ A_6 \succ A_2$
$\lambda = 0.6$	0.029	0.015	0.025	0.025	0.049	0.013	0.041	$A_5 \succ A_7 \succ A_1 \succ A_3 \succ A_4 \succ A_2 \succ A_6$
$\lambda = 0.7$	0.053	0.027	0.045	0.045	0.079	0.022	0.070	$A_5 \succ A_7 \succ A_1 \succ A_4 \succ A_3 \succ A_2 \succ A_6$
$\lambda = 0.8$	0.077	0.039	0.064	0.065	0.110	0.032	0.099	$A_5 \succ A_7 \succ A_1 \succ A_4 \succ A_3 \succ A_2 \succ A_6$
$\lambda = 0.9$	0.101	0.051	0.084	0.085	0.140	0.041	0.128	$A_5 \succ A_7 \succ A_1 \succ A_4 \succ A_3 \succ A_2 \succ A_6$
$\lambda = 1$	0.124	0.063	0.103	0.105	0.170	0.050	0.157	$A_5 \succ A_7 \succ A_1 \succ A_4 \succ A_3 \succ A_2 \succ A_6$

Table 8
Sensitivity analysis for λ value for $\widetilde{D}_{IFWG}$.

Score of $\widetilde{D}_{IFWG}$	A_1	A_2	A_3	A_4	A_5	A_6	A_7	Ranking
$\lambda = 0$	−0.107	−0.070	−0.050	−0.078	−0.192	−0.064	−0.217	$A_3 \succ A_6 \succ A_2 \succ A_4 \succ A_1 \succ A_5 \succ A_7$
$\lambda = 0.1$	−0.083	−0.058	−0.043	−0.064	−0.160	−0.056	−0.186	$A_3 \succ A_6 \succ A_2 \succ A_4 \succ A_1 \succ A_5 \succ A_7$
$\lambda = 0.2$	−0.058	−0.046	−0.036	−0.050	−0.128	−0.049	−0.155	$A_3 \succ A_2 \succ A_6 \succ A_4 \succ A_1 \succ A_5 \succ A_7$
$\lambda = 0.3$	−0.034	−0.033	−0.028	−0.036	−0.097	−0.041	−0.125	$A_3 \succ A_2 \succ A_1 \succ A_4 \succ A_6 \succ A_5 \succ A_7$
$\lambda = 0.4$	−0.009	−0.021	−0.021	−0.022	−0.065	−0.031	−0.094	$A_1 \succ A_3 \succ A_2 \succ A_4 \succ A_6 \succ A_5 \succ A_7$
$\lambda = 0.5$	0.015	−0.009	−0.014	−0.008	−0.033	−0.025	−0.063	$A_1 \succ A_4 \succ A_2 \succ A_3 \succ A_6 \succ A_5 \succ A_7$
$\lambda = 0.6$	0.0340	0.003	−0.007	0.005	−0.001	−0.017	−0.032	$A_1 \succ A_4 \succ A_2 \succ A_5 \succ A_3 \succ A_6 \succ A_7$
$\lambda = 0.7$	0.064	0.015	0.001	0.019	0.030	−0.009	−0.002	$A_1 \succ A_5 \succ A_4 \succ A_2 \succ A_3 \succ A_7 \succ A_6$
$\lambda = 0.8$	0.089	0.028	0.008	0.033	0.062	−0.001	0.029	$A_1 \succ A_5 \succ A_4 \succ A_7 \succ A_2 \succ A_3 \succ A_6$
$\lambda = 0.9$	0.113	0.040	0.015	0.047	0.094	0.007	0.060	$A_1 \succ A_5 \succ A_7 \succ A_4 \succ A_2 \succ A_3 \succ A_6$
$\lambda = 1$	0.138	0.052	0.022	0.061	0.126	0.015	0.090	$A_1 \succ A_5 \succ A_7 \succ A_4 \succ A_2 \succ A_3 \succ A_6$

- **Step 7:** The score values are obtained by Eq. (14) and Eq. (15) as in Table 10. The ranking of the alternatives is $A_5 \succ A_7 \succ A_3 \succ A_1 \succ A_4 \succ A_6 \succ A_2$ from $\widetilde{D}_{IFWA}$ and $A_1 \succ A_4 \succ A_2 \succ A_3 \succ A_6 \succ A_5 \succ A_7$ from $\widetilde{D}_{IFWG}$.

Comparing the IFS MCDM and the proposed C-IFS MCDM procedures, the results in the first part, "Section 4.1 Deriving criteria weights", are close. The rankings of criteria are similar in both procedures. Small differences are normal because the decision-maker uncertainty expressed by the radius present in the proposed C-IFS model has not been taken into account. In the second part, Section 4.2, alternatives are ranked by using the criteria weights from the first part. The aggregated decisions $\widetilde{D}$ for both IF and C-IFS MCDM procedure are very similar. However, since there is no radius and lambda value in the IFS MCDM procedure, the aggregated decisions $\widetilde{D}$ are defuzzified and the result is reached. Here, since the lambda value (λ) reflects the decision-maker's attitude and cannot be applied in IFS MCDM, it would be quite appropriate to interpret the lambda value as 0.5 ($\lambda = 0.5$) in the C-IFS MCDM results to compare the two procedures. Hence, Table 11 summarizes the results of the two approaches.

5. Results and discussion

To overview the proposed approaches, it is started with defining the case and choosing the appropriate experts to the subject by managers. It is important to make the choice, as decision-makers may have different perspectives and experiences on the subject. The weights were taken into account for the person managing the process to rate the experts. It is also the decision of the project manager to see all experts as equal and it is appropriate to accept the weights equal to the proposed model. As a second step, decision-makers evaluate the case by using defined IF linguistic terms.

For criteria weighting, in the third step, four alternatives aggregated IF decision matrix are emerged using Eqs. (4)-(7) as in Table 3. Although the matrix results are quite similar, the calculation method involves some variation. Eq. (4) ignores the decision-maker weights. In this formula proposed by Atanassov, a new central IFS is found with an arithmetic mean among the centers of the decision-makers' IF evaluations. In MCDM, this makes sense in the absence of weights of decision-makers. Therefore, this study proposed to use other formulations that take into account decision-maker weights. Eq. (5) gives the weighted average of the centers of the IF numbers. Similarly, based on the fact that these numbers are IF numbers, a new number to be obtained from these numbers can also be obtained with the help of operators. For this, the IFWA and IFWG operators, which were previously defined in the literature, are implemented using Eq. (6) and Eq. (7).

In cases involving weights, operations can be performed using operators. The purpose of all formulas in Eqs. (4)-(7) is to calculate the center point of new C-IFS numbers that will contain all the decisions for the aggregated matrix. Then, in step four, Eq. (8) helps to find the radiuses of these new C-IFS numbers, which includes all IF decisions. The radius size is related to the uncertainty of the C-IFS number. A larger radius indicates that decisions are more dispersed, as they form a larger circle. For example, in the case, the radius of C1 is 0.35, while the radius

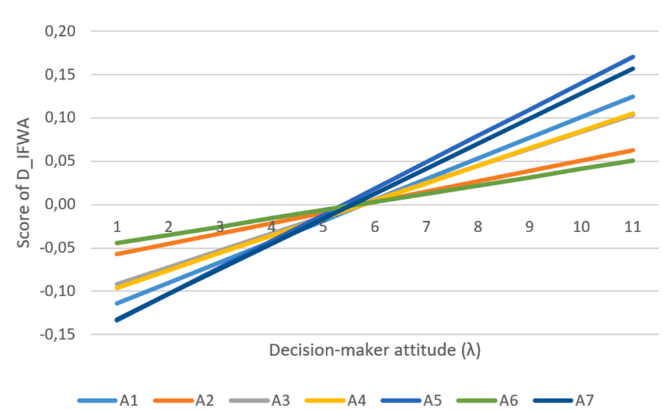

Fig. 10. Change in alternative rankings by λ value for $\widetilde{D}_{IFWA}$.

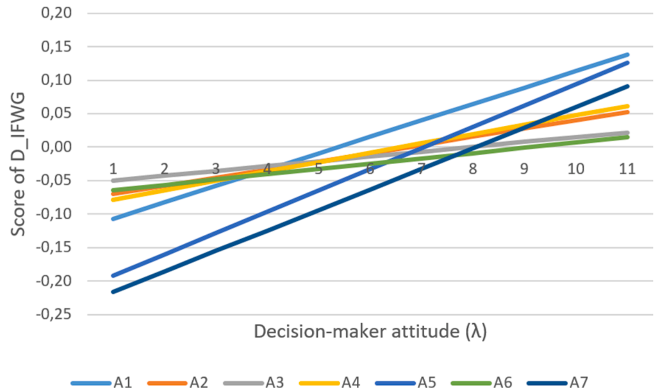

Fig. 11. Change in alternative rankings by λ value for $\widetilde{D}_{IFWG}$.

Table 9
Normalized criteria weights of IFS MCDM procedure.

Criterion	$S_{IFS}(C_i)$	Normalized Score
C_1	−0,04	0.20
C_2	−0,27	0.15
C_3	−0,31	0.14
C_4	0,39	0.29
C_5	0,01	0.21

of C3 is 0.23. This reveals that the decision-makers made a more approximate decision in criterion 3. While the center of the C-IFS number indicates the membership or non-membership status of the criterion; the radius shows the agreement of the decision-makers. The fifth step defuzzifies the C-IFS weights of the criteria with the given formula. Note that the defuzzification formula is obtained from Eq. (18) by setting the λ as 1 and the range as [0–1]. The importance of the criteria for supplier selection is ranked as $C_4 \succ C_5 \succ C_2 \succ C_1 \succ C_3$.

For alternative ranking according to the criteria, the first five steps are similar to criteria weighting. Instead of the decision-maker weight, the criterion weight is included in the calculation. Since many MCDM models do not take the criteria weights equally, it is more convenient to use the IFWA and IFWG operators to find the C-IFS centers at this stage. The C-IFS values are similar in the $\widetilde{D}_{IFWA}$ and $\widetilde{D}_{IFWG}$ matrices. Eq. (18) is performed while defuzzifying the C-IFS values of the alternatives. This formula includes the manager's optimistic or pessimistic view for the subject. For example, λ is taken as 0.7 in the evaluation suppliers. This allows the administrator to proceed from the point where the membership degrees of the C-IFS numbers are higher and non-membership degrees are lower in the calculations. Similarly, λ being close to zero causes C-IFS numbers to be calculated from the side with less membership degrees and more non-membership degrees, which is a pessimistic point of view. Table 7 and Table 8 show the rankings for all, λ values for both operators.

For each λ, the ranking of the alternatives with scores of $\widetilde{D}_{IFWA}$ and $\widetilde{D}_{IFWG}$ are given in Table 7. It is clear that the three best alternatives are A1-A5-A7 and the last three alternatives are A2-A3-A6. A4 is generally in the middle of the ranking. λ value close to 0 brings undesirable alternatives to the top, this is due to the manager's pessimistic point of view. Conversely, as the λ approaches 1, it maximizes the last-ranked alternatives as the most desirable alternatives when the λ is at 0.

Comparing the results with the literature, it is clear that the results from the IFS MCDM approach, which has been proven in the literature and used in many applications, are consistent with the C-IFS MCDM approach which is proposed in this article. The lambda value (λ), which indicates the decision-maker's attitude, and the radius value (r), which expresses the uncertainty of the decisions, in the novel model create changes in the results. However, in line with the information obtained from Table 7 and Table 8, the existence of these parameters will enable the establishment of a structure that can change according to the case, and which does not include IFS MCDM approaches. This diversity makes it meaningful to use C-IFS numbers in MCDM models.

Table 10
Scores of alternatives of IFS MCDM procedure.

Alternative	Score of $\widetilde{D}_{IFWA}$	Score of $\widetilde{D}_{IFWG}$
A_1	0.016	0.046
A_2	0.008	−0.027
A_3	0.0017	−0.042
A_4	0.0014	−0.025
A_5	0.056	−0.099
A_6	0.009	−0.074
A_7	0.037	−0.189

6. Conclusion

This study proposes new C-IFS MCDM procedures in criteria weighting and alternative ranking by proposing novel functions, which are score (S_{C-IFS}) and accuracy (H_{C-IFS}) functions. It also describes new perspectives in radius calculation and the inclusion of interior points in defuzzification. To analyze the proposed C-IFS MCDM methodology, alternative results of the supplier selection using all suggested formulas are given together. A supplier selection problem for a seamless supply chain network as a case study illustrates the steps of procedures. The proposed fuzzy methodology is aimed to assist the decision making processes of organizations within various industries. Sensitivity analysis is performed according to the parameter (λ) changes, and the results of two operators (IFWA and IFWG) are compared.

Although the presented methods contribute to the fuzzy literature by adapting C-IFS on MCDM, they also have some limitations. The determination of decision-maker groups varies according to the cases. Weighting decision-makers or the manager's point of view for λ parameters affects the results. The equations used in the radius calculation depend on the discretion of the person who applies the method. The larger the radius, the greater the uncertainty in the expression. Another limitation is that although the score function ($S_{C-IFS}(c)$) in Eq. (18) depends on the radius (r) and the decision-maker's attitude (λ), the accuracy function ($H_{C-IFS}(c)$) in Eq. (19) is independent of them. At this point, it is clear that the accuracy function calculates as IF numbers. Therefore, it is not recommended to use the accuracy function alone when sorting C-IFS. In case of equality after applying Eq. (18), it would be more convenient to use Eq. (19). Also, aggregation operators preferred from different perspectives cause minor changes in ordering. Considering the C-IFS as an extension of IFS, the similarity of the results obtained with the IFS MCDM procedure also proves the usability of the proposed C-IFS MCDM algorithm.

For further researches, the proposed circular intuitionistic fuzzy MCDM approach can be used in application areas, such as human

Table 11
Comparison of results for IFS MCDM and C-IFS MCDM procedure.

	C-IFS MCDM	IFS MCDM
Criteria Ranking	$C_4 \succ C_5 \succ C_2 \succ C_1 \succ C_3$	$C_4 \succ C_5 \succ C_1 \succ C_2 \succ C_3$
Alternative Ranking with *IFWA*	$A_5 \succ A_7 \succ A_3 \succ A_1 \succ A_4 \succ A_6 \succ A_2 (\lambda = 0.5)$	$A_5 \succ A_7 \succ A_3 \succ A_1 \succ A_4 \succ A_6 \succ A_2$
Alternative Ranking with *IFWG*	$A_1 \succ A_4 \succ A_2 \succ A_3 \succ A_6 \succ A_5 \succ A_7 (\lambda = 0.5)$	$A_1 \succ A_4 \succ A_2 \succ A_3 \succ A_6 \succ A_5 \succ A_7$

resources evaluation, risk assessment, location selection, technology and investment selection, where evaluation statements containing uncertainty in an incomplete and vague information environment. The supplier selection problem for a seamless supply chain network discussed in the case study can be reconsidered from various perspectives (sustainable, resilient, digital, and so on). In addition, to enlarge its usage, the results can be evaluated by comparing the proposed methodology with other standardized well-known MCDM models including AHP, ANP, TOPSIS and VIKOR with many conflicting criteria to consider in solving real-life problems.

CRediT authorship contribution statement

Esra Çakır: Methodology, Writing – original draft. **Mehmet Ali Taş:** Writing – review & editing.

Declaration of Competing Interest

The authors declare the following financial interests/personal relationships which may be considered as potential competing interests: ESRA CAKIR reports financial support was provided by Galatasaray University.

Data availability

No data was used for the research described in the article.

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