

## Designing robust capability-based distributed machine layouts with random machine availability and fuzzy demand/process flow information

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### Abstract

Thus far in the available literature, capability-based distributed layout (CBDL) design approaches were only developed under certain environments. Indeed, uncertainties embedded in the machine unavailability (or random machine breakdowns), product demands, and process flow data were not considered by the previous studies to achieve a robust CBDL design. However, many real-life facility layout design applications may involve different types of uncertainties simultaneously, like fuzziness and stochasticity. Based on this motivation, for the first time in the literature, this paper introduces a novel robust capability-based distributed layout (R-CBDL) design problem under a mixed fuzzy-stochastic environment. First, a new fuzzy-stochastic optimization model of the R-CBDL design problem is developed by considering the random machine breakdowns and fuzzy demand/process flow data. Then, a hybrid solution approach based on a chance-constrained stochastic programming technique with a well-known interactive fuzzy resolution method is proposed. Thus, the random machine breakdowns and fuzzy part flow rates among different machining capabilities could be easily handled via the proposed approach. Fortunately, the proposed approach can also generate various risky and risk-free robust layout design alternatives under different probabilistic scenarios and uncertainty levels ( $\alpha$ -cuts) according to the facility designer's risk attitude. To show the validity and applicability of the proposed R-CBDL problem and hybrid solution approach, an extensive computational study with comparative analysis is first presented based on an illustrative numerical example under different machine capability overlap cases and probability distributions. Then, the performance of the proposed approach is also tested on a real-life cellular manufacturing system of a company. The computational experiments have shown that the proposed approach can accomplish more efficient robust layout design alternatives with on average 24.5% better total expected layout score when compared to the existing cellular layout of the manufacturing company.

**Keywords** Robust capability-based distributed layout problem  $\cdot$  Chance-constrained stochastic program  $\cdot$  Fuzzy mathematical programming  $\cdot$  Random machine availability  $\cdot$  Fuzzy part flows

## 1 Introduction

Although the facility layout design (FLD) concept is a highly studied research topic, many studies on its different variants have still been conducted in the recent literature

Kemal Subulan kemal.subulan@gmail.com (Guo et al. 2022; Baykasoğlu et al. 2022; Yang and Lu 2023; Subulan et al. 2023; Pérez-Gosende et al. 2023; Zolfi et al. 2023). The CBDL design approach that was first proposed by Baykasoğlu (2003) and then also extended in recent studies (Baykasoğlu and Subulan 2020; Baykasoğlu et al. 2022; Subulan et al. 2023) is one of its variants which aim to distribute machines' processing capabilities properly over the facility floor to adapt dynamically changing manufacturing environments. In the original approach of Baykasoğlu (2003), machines' processing capabilities which are defined in terms of Resource Elements (REs) were considered as the basic capability units and optimal distribution of these REs was targeted to provide

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distributed layouts. When compared to the classical machine-based distributed layout design approaches (where the optimal distribution of machines is considered only), the capability-based distribution was proven to be a more effective option as it can also reveal the hidden flexibility in the manufacturing systems (Baykasoğlu and Göçken 2010; Baykasoğlu and Subulan 2020; Baykasoğlu et al. 2022). Since some processing capabilities can be covered by alternative machines, the capability-based definition for the processing requirements of the manufactured parts may offer more flexibility than a machinebased route definition where a part processing requirement is generally assigned to a single machine. Furthermore, in many real-world manufacturing environments, parts' processing requirements (or manufacturing routes) are commonly described in terms of the production operations (or machine's processing capabilities/REs) they need. Based on these facts, several subsequent papers have been also conducted on the CBDL design approach (Hamedi et al. 2012a,b; Hamedi and Esmaeilian 2015; Shafigh et al. 2015, 2017; Baykasoğlu and Subulan 2020; Baykasoğlu et al. 2022; Subulan et al. 2023). Actually, the original approach of Baykasoğlu (2003) was first extended to a biased CBDL design approach by Baykasoğlu and Subulan (2020) by considering demand data and process flow information, because some dominant part flow patterns may arise among different machining capabilities in realworld settings and this may also affect the optimal capability distribution over the facility floor. It was also highlighted by Benjaafar and Sheikhzadeh (2000) and Benjaafar et al. (2002) that if available, process flow information and demand data may lead to better-quality distributed layouts. Therefore, dominant part flow patterns among different machining capabilities could be taken into account with the help of the biased CBDL approach of Baykasoğlu and Subulan (2020). The latest variant of capability-based FLD approaches which is named as unequal-area capability-based facility layout design (UA-CBFLD) was recently introduced by Subulan et al. (2023). However, all of these previous studies on the capabilitybased FLD approaches (Baykasoglu 2003; Baykasoğlu and Göcken 2010; Baykasoğlu and Subulan 2020; Baykasoğlu et al. 2022; Subulan et al. 2023) were only proposed under certain environments. In other words, uncertainties embedded in the product demand, process flow data, and machine unavailability risks (or breakdowns) were not considered by these former studies to generate a robust capability-based FLD.

It should also be highlighted here that although several variants of the robust FLD approaches are available in the

literature (Rosenblatt and Lee 1987: Aiello and Enea 2001: Pillai et al. 2011; Moslemipour et al. 2012, 2017; Drira et al. 2013; Nematian 2014; Neghabi et al. 2014; Izadinia et al. 2014; Fazlelahi et al. 2016; Zha et al. 2017, 2020; Morinaga et al. 2017, 2019; Peng et al. 2018; Targhi et al. 2019; Xiao et al. 2019; Vitayasak et al. 2019; Hunagund et al. 2020; Khajemahalle et al. 2021; Esmikhani et al. 2022), there is still a lack of studies on the robust distributed layout design approaches (both on machine-based and/or capability-based ones) under uncertain environments. In other words, robust distributed layout approaches were rarely discussed in the existing literature (Celik et al. 2016; Pourvaziri et al. 2022). Moreover, even though many studies are also available on different variants of the FLD problems with uncertain demand and/or process flow data, the number of studies focusing on the FLD problems with random machine breakdowns (or unavailability risks) is very limited in the existing literature (Vitayasak and Pongcharoen 2016; Vitayasak et al. 2019). In other words, the majority of robust FLD approaches (Vitayasak et al. 2017; Zha et al. 2020; Khajemahalle et al. 2021; Gölcük et al. 2022; Pourvaziri et al. 2022) have concentrated on the uncertainty of product demands, process flows, or closeness ratings only. However, there are still a few studies on the FLD approaches with stochastic machine unavailability or breakdowns. On the other hand, random machine breakdowns that correspond to machine unavailability risks may have a considerable effect on the layout design objectives (Vitayasak and Pongcharoen 2016; Vitayasak et al. 2019). It should also be noted here that none of these aforementioned studies on the robust FLD problems took into account the machines' processing capabilities to produce a robust capability-based layout design. In other words, the machines' processing capabilities and their optimal distribution over the facility floor were not considered by the previous studies on robust FLD problems. Furthermore, many real-world problems may contain different types of uncertainties such as fuzziness and randomness (or stochasticity) together (Baykasoğlu and Subulan 2019; Subulan 2020; Subulan and Çakır, 2022). Therefore, different types of uncertain parameters may also arise in real-world facility layout design applications. However, only one type of uncertainty, i.e., either fuzziness or stochasticity was considered by the majority of the previous studies (Kulturel-Konak 2007; Nematian 2014; Moslemipour et al. 2018; Pagone et al. 2023) while handling the non-deterministic facility layout parameters. However, these different uncertainty categories (i.e., fuzziness, randomness, and hybrid or fuzzy random) should be addressed together for better reflection of real-life settings (Liu 2007).

Based on these motivations, the main contributions of this study are presented as follows:

- (i) This paper introduces a novel robust capabilitybased distributed layout (R-CBDL) design problem under a mixed fuzzy-stochastic environment.
- (ii) Unlike the previous studies on machine-based robust distributed layout design approaches (Celik et al. 2016; Pourvaziri et al. 2022), we introduced an R-CBDL design approach for the first time in the literature. To the best of our knowledge, there is no similar study in the literature on such an R-CBDL approach that can consider random machine breakdowns and fuzzy demand/process flow information simultaneously.
- (iii) In addition to the uncertainties in the demand quantities and process flow data of several parts, we also took into account the random machine breakdowns (or machine unavailability risks) to provide a more effective robust CBDL. Because the random machine breakdowns may affect the accessibility of the machines' processing capabilities considerably and may also have an influence on the part flows among different machines. Indeed, flow patterns between different machining capabilities, which can be obtained from the processing routes of several parts, may not generally be determined precisely in many real-world applications due to fuzzy demand quantities and process flow data. For these reasons, as in the studies of Vitayasak and Pongcharoen (2016) and Vitayasak et al. (2019), the proposed R-CBDL approach aims to assign alternative machines, which is not broken down and also have the same processing or machining capabilities. As mentioned previously, none of these former studies on robust FLD problems considered the optimal distribution of machines' processing capabilities. Fortunately, we took into account these processing capabilities which can also provide additional flexibility in case of machine breakdowns. In other words, alternative machines that are not broken down and have the same processing capabilities can be used instead of broken or inactive ones. However, it should also be emphasized here that it is not always possible to find a feasible FLD under some probabilistic scenarios where some critical machines are broken down. In this case, critical machines which necessitate preventive maintenance actions can also be easily specified by using the unsatisfied scenarios in the proposed hybrid solution approach (which is based on chanceconstrained stochastic programming and fuzzy

interactive resolution techniques). To the best of our knowledge, there is no similar study on the robust FLD approaches which can specify the critical machines (with the most essential processing capabilities) that require a preventive maintenance plan.

(iv) For better reflection of the real-life applications which generally contain different types of uncertainties simultaneously, we first formulated a new fuzzy-stochastic optimization model of the proposed R-CBDL design problem based on the original deterministic MILP model of Baykasoğlu and Subulan (2020). Then, a hybrid solution approach is proposed based on a chance-constrained stochastic program and a well-known interactive fuzzy resolution method. In addition to coping with different types of uncertainties concurrently, the proposed hybrid solution approach is also able to specify the critical machines with the most vital processing capabilities by using the unsatisfied scenarios in the chance constraint sets. Thus, facility designers can be aware of these critical machines and aim to increase their reliability rates (or decrease their breakdown probabilities) to maintain the continuous production of the facilities. Furthermore, the proposed solution approach can also generate various layout design alternatives under different probabilistic scenarios and uncertainty levels ( $\alpha$ -cuts or feasibility degrees of fuzzy constraints) concerning the facility designer's risk attitude (i.e., risk-averse or seeker). Therefore, more reliable and efficient robust distributed layout design alternatives can be generated via the proposed approach. To the best of our knowledge, there is no similar study in the literature, which can produce various robust layout design alternatives under different scenarios and uncertainty levels according to the facility designer's attitude toward risk.

The remainder part of this paper is organized as follows: A brief literature review on robust FLD problems under uncertainty is presented in Sect. 2. The description and mathematical formulation of the R-CBDL design problem are given in Sect. 3. In Sect. 4, the fundamentals of the proposed hybrid approach are discussed in detail. The proposed approach is first illustrated on a basic numerical example in Sect. 5. In that section, a comparison of the deterministic, stochastic, fuzzy, and fuzzy-stochastic optimization results is also presented by considering different machine capability overlap cases and probability distributions. In Sect. 6, the performance of the proposed approach

is tested on a real-life problem. Finally, conclusions and future research are discussed in Sect. 7.

### 2 Literature review on robust FLD problems under uncertainty

In this section, a brief literature survey on robust FLD problems under uncertainty is presented. In fact, there are two main sources of uncertainties in many real-world facility layout design applications, namely (i) the uncertain product demand (or process flow data) and (ii) machine unavailability risks (or reliability) due to random machine breakdowns. Under highly volatile manufacturing environments, these uncertainties are essential concerns for designing robust facility layouts in manufacturing industries since they may affect the performance of the manufacturing systems significantly (Kulturel-Konak 2007; Neghabi et al. 2014; Salimpour et al. 2021; Esmikhani et al. 2022; Pagone et al. 2023). As also highlighted by Liu (2007), fuzzy, random, and hybrid (i.e., fuzzy-random) variables are three main instances in uncertainty theory. In fact, fuzzy and random variables are special cases of hybrid variables. Furthermore, it was also noted by Liu (2004, 2007) that probability measure (i.e., for random variables) and credibility measure (i.e., for fuzzy variables) are special cases of chance measure (i.e., for hybrid variables) and three of them are in the category of uncertain measure. The uncertainty relations between these uncertain variables and measures were also summarized by Liu (2007). In the first part of this literature survey, we concentrated on the robust FLD in which uncertain product demand and process flow data are considered. Afterward, we reviewed the articles that focused on random machine breakdowns, which correspond to machine unavailability risks. Furthermore, articles that focused on uncertainties other than demand (or process flow) and machine breakdowns are also reviewed in the last part of this survey.

An overview of the articles which dealt with the uncertain product demand and/or process flow data can be summarized as follows: One of the pioneering studies was performed by Rosenblatt and Lee (1987) on a robustness approach for a single period plant layout problem under stochastic demand data. They targeted to generate robust layout alternatives that perform well under many different scenarios. In a similar way, Aiello and Enea (2001) also proposed a fuzzy approach to the robust FLP where the market demand was represented by triangular fuzzy numbers. Ertay et al. (2006) introduced a robust layout framework that is based on the integration of data envelopment analysis (DEA) and analytic hierarchy process (AHP) methodologies. Furthermore, they handled demand variability by means of fuzzy linguistic variables while

gathering data related to the activity relationships for a computer-aided layout-planning tool, i.e., VisFactory. Kulturel-Konak (2007) discussed the most recent advancements and approaches to model and analyze uncertainties in FLD problems. She has presented a detailed review and classification of all of the former studies in this field. She also highlighted that inclusion of the uncertainty in FLD models is very essential for their applicability in real-life settings. This is because nowadays, business economies become even more volatile and product life cycles shorten continuously. Therefore, the configuration of robust and flexible facility layouts under such dynamic and uncertain conditions is vital for both manufacturing and service industries. Moslemipour et al. (2012) also presented a detailed review of intelligent approaches along with their advantages and disadvantages for designing dynamic and robust layouts in flexible manufacturing systems. A simulation-based approach was developed by Jithavech and Krishnan (2010) for a risk-based FLD problem under stochastic product demand. They first utilized the simulation method for risk evaluation and prediction of the uncertainty, and then, a genetic algorithm (GA) was designed to produce the layout alternatives for both the forecasted demand case and the risk-based layout design. Pillai et al. (2011) developed a simulated annealing (SA)-based metaheuristic algorithm to design a robust layout for the dynamic plant layout problem under a dynamic demand environment. They proposed a robust layout procedure to cope with this dynamic environment and produce layout alternatives with minimum material handling and relocation costs for an expected demand scenario. A GA-based evolutionary fuzzy approach was developed by Drira et al. (2013) for solving a dynamic layout problem with increasing uncertainty in customer demands over time. The production demand of each part was defined by using triangular fuzzy numbers. They also concluded that degradation of the available information on customer demands may significantly impact the facility layout, and therefore, demand uncertainty has to be considered if a robust layout is required. A mixed-integer nonlinear programming model was proposed by Izadinia et al. (2014) for a robust multi-floor layout problem (MFLP) in which material flows among the departments and storages were changed in the pre-specified intervals. Vitayasak and Pongcharoen (2015b) aimed to analyze the impact of demand variation over time periods on a robust machine layout design problem in which total material handling distance was minimized by a GA-based metaheuristic approach. A Comparison of the re-layout and robust layout approaches was also carried out by Vitayasak and Pongcharoen (2015a) for a non-identical machine layout problem under stochastic demand. A greedy heuristic procedure was developed by Zhao and Wallace

(2015) for solving a single-product capacitated FLD problem which aims to minimize total expected material handling cost under uncertain demand. Actually, they assumed that the product demand varies according to a known continuous probability distribution from period to period. By using this heuristic approach, the stochastic FLD problem was converted into a traditional quadratic assignment problem (QAP). Celik et al. (2016) proposed a dynamic programming-based heuristic procedure for solving a multi-period stochastic distributed FLD problem, which aims to determine the relative locations of the multiple copies of capacitated machinery. The dynamic and stochastic nature of demand and the possibility of relayouts were also considered while minimizing the expected total material handling cost. An integrated permutation-based GA and robust optimization technique was introduced by Fazlelahi et al. (2016) for the dynamic facility layout problem with demand uncertainty. They aimed to produce a unique robust layout plan for all of the time periods. A position-based flexible particle swarm optimization (PSO) algorithm was proposed by Zha et al. (2017) to solve a robust dynamic facility layout model with unequal-area departments. They took into account the uncertain product demands, which are represented by fuzzy random variables. Moslemipour et al. (2017) proposed a novel mathematical model based on the QAP formulation for designing a dynamic robust facility layout. They supposed that the periodic product demands are normally distributed with known parameters, i.e., mean, standard deviation, and covariance, which vary randomly from one period to another. A hybrid algorithm that integrates the SA algorithm, ant colony, clonal selection, and robust layout design approaches was employed by Moslemipour et al. (2018) to solve a stochastic dynamic facility layout problem where product demands were defined as normally distributed random variables. Peng et al. (2018) proposed a mathematical programming model, which also considers transport device assignment for the stochastic dynamic facility layout problem. In order to generate multiple demand scenarios randomly, they utilized the Monte Carlo simulation approach. A scenario-based stochastic optimization model was proposed by Sahinkoç and Bilge (2018) depending on the classical QAP formulation in which part flows between the departments are uncertain. After transforming this scenario-based uncertainty into a multi-objective optimization model, they developed a multi-objective evolutionary algorithm to find the Pareto optimal set regarding different robustness performance measures. A robust optimization model was developed by Xiao et al. (2019) for an unequal-area dynamic FLD problem with demand and logistics uncertainties. In order to deal with the dynamic changes in product demand and mix, Pourvaziri et al. (2022) developed an approach that decomposes the robust facility layout design problem into two sub-problems. In the first sub-problem, they constructed a robust layout by using a design-of-experiment (DOE) approach, and then, a branch-and-cut algorithm was applied to solve the second sub-problem to obtain the best routes of products in each time period. A hybrid algorithm based on the Nested Partitions (NP) and SA algorithms was designed by Khajemahalle et al. (2021) for the robust dynamic facility layout problem. They assumed that the material flows between departments and rearrangement costs are uncertain parameters whose values varied in the symmetric range of intervals. A new hybrid MADM model that is based on the interval type-2 fuzzy-full consistency method (IT2F-FUCOM), interval type-2 fuzzy activity relationship charts (IT2F-ARCs), and the measurement alternatives and ranking according to the compromise solution (MARCOS) method was developed by Gölcük et al. (2022) for evaluating facility layout alternatives.

Although there are many studies on the robust FLD problems with uncertain (i.e., fuzzy or stochastic) demand and process flow data, the number of studies focusing on the uncertainty in machine unavailability risks (or breakdowns) is very limited in the literature. In the literature on FLD problems, the articles that considered the uncertainties in machine breakdowns with maintenance issues are summarized as follows: Vitayasak and Pongcharoen (2016) stated that machine breakdown is a stochastic or random event, which is an essential concern for designing facility layouts in manufacturing industries. Therefore, they presented a GA application for quantifying the effect of breakdown maintenance on the performance of machine layouts. They assumed that an alternative machine may be assigned to perform the required manufacturing operations if a machine is broken down in real-world settings. This also affects the material handling distance since the route of the machine sequence is changed. Similarly, in the present paper, we also aim to provide a robust CBDL by assigning alternative machines, which have the same processing capabilities (or REs) as well as not broken down. The study of Vitayasak and Pongcharoen (2016) was also extended by Vitayasak et al. (2019) to incorporate dynamic demand and machine maintenance planning options (i.e., preventive/corrective) for designing non-identical machine layouts. They also implemented the GA approach to analyze the effects of these maintenance strategies under different demand distributions. They concluded that robust layout designs which considered the random machine breakdowns and preventive maintenance strategy may cause lower costs in breakdown maintenance conditions and also lead to the greatest reduction in material flow distances when compared to the re-layout design approaches. Based on this motivation, this paper proposes a robust CBDL approach that is also able to specify critical machines with the most important processing capabilities. Actually, we used the unsatisfied chance constraint sets to determine these critical machines. Thus, more reliable and efficient robust distributed layout alternatives can be obtained by predetermining these critical machines, which necessitate a preventive maintenance plan.

The FLD studies focusing on the other uncertainty issues rather than the demand (or process flow) and machine breakdowns are also summarized as follows: Meller and Gau (1996) examined the adjacency-based, distance-based, and weighted criteria facility layout objectives to design robust layouts. Since specifying certain weights may not be an exact process, they employed a discrete efficient frontier method with imperfect weighting value information. An exact algorithm based on a dynamic adaptive iterative procedure was developed by Neghabi et al. (2014) for solving a robust multi-row FLP where the size of each department is stated as uncertain. They defined permissible intervals for the length and width of each department. Nematian (2014) introduced a robust singlerow facility layout model with fuzzy random variables. The cost of transmitting flows between departments, the length of departments, and the minimum gap between departments were represented by LR fuzzy numbers. After transforming the robust model into its crisp equivalent form, a new exact algorithm based on an extended branchand-bound algorithm was applied to achieve optimal solutions for several test problems. A bi-objective MIP model was developed by Salmani et al. (2015) for solving a robust multi-row facility layout problem with dynamic and uncertain departments' dimensions where each dimension changes in a pre-determined interval. Morinaga et al. (2017) introduced a robust facility layout planning method by considering temporal efficiency and routing. For enabling robustness, they assumed that the sum of products varies over production scenarios with known probabilities. Thus, an additional evaluation index has been specified to obtain facility layouts by considering numerous scenarios. They applied a simulation optimization approach with GA to perform production scheduling within a reasonable time since this additional index causes significant increases in computational effort. In order to reduce this computational load, Morinaga et al. (2019) also developed a new method based on a sampling approach and GA. In that method, individuals are evaluated based on the mean fitness in some scenarios that are sampled randomly, and the selection process is performed by using Welch's test. Unlike the existing studies in the literature, Targhi et al. (2019) developed a mathematical programming model for a robust three-dimensional and multi-floor facility layout problem. Since the parameters of this problem can be changeable, they utilized robust planning in overlapping constraints. Hunagund et al. (2020) also developed a MILP model for designing a robust unequal area facility layout with a flexible bay structure under a dynamic environment. Esmikhani et al. (2022) proposed a multi-objective population-based SA algorithm (MPS) and a Modified Non-dominated Sorting Genetic Algorithm (MNSGA-II) to solve a fuzzy robust facility layout problem equipped with cranes. In addition to the material flows between facilities, they assumed that the facility dimensions are hybrid uncertain parameters and stated as fuzzy random variables.

Most of the previously discussed studies are summarized in Table 1 according to the problem type and characteristics (i.e., problem objectives, equal or unequal department areas, type of uncertainty: fuzziness and/or stochasticity, uncertain components/parameters) and the solution methodology. As it is clearly seen in Table 1, the majority of available robust FLD approaches concentrated on the uncertainty of product demand or process flow only. However, there are still a few studies on the FLD approaches with machine unavailability risks (or breakdowns). Actually, this is a research gap in the literature on robust FLD problems. Moreover, none of the aforementioned studies considered the machines' processing capabilities that are defined in terms of REs. Fortunately, consideration of these machining capabilities may provide additional flexibility in case of machine breakdowns. In other words, alternative machines that are not broken and have the same processing capabilities can be used instead of broken or inactive machines. It should also be noted here that although several robust FLD approaches are available in the literature, there is still a lack of studies on robust distributed layout design approaches under uncertainty. In other words, robust distributed layouts were rarely discussed in the existing literature. Finally, only one type of uncertainty, i.e., either fuzziness or stochasticity (randomness) was considered by most of the previous studies existing in the literature. However, different types of uncertainties may generally arise simultaneously in many real-life FLD problems. For that reason, these different uncertainties should be considered together while generating robust facility layouts.

Based on these motivations, this paper also introduces a hybrid solution approach based on a chance-constrained stochastic program and an interactive fuzzy resolution method for solving the proposed robust CBDL problem with random machine availability and fuzzy demand/process flow information. We also need to mention here that we have considered both fuzzy (i.e., fuzzy demand quantities/process flow data) and random variables (i.e., random machine unavailability or breakdowns) but not considered a hybrid uncertain variable in this paper. In other words, some uncertain parameters are defined as fuzzy (i.e., demand quantities and therefore, part flows), whereas the other uncertain parameter (i.e., machine breakdown or

Article	Problem type	Problem characteristics	aracteri	stics		[dO	Objectives	Solution methodology
		Type of area		Uncertainty	Uncertain components			
		Equal Une	Unequal	Fuzzy Stochastic	Demand Breakdown	Other		
Rosenblatt and Lee (1987)	Single-period plant layout problem	7		2	7	Mii hî	Minimum total material handling costs	Robustness approach
Meller and Gau (1996)	Robust FLP	7		7	7		Minimum total material handling costs	Robust layout method and discrete efficient frontier method
Aiello and Enea (2001)	Robust FLP	7		7	7	Min hí	Minimum total material handling costs	Constrained arithmetic operator and fuzzy ranking method
Ertay et al. (2006)	Robust FLP	7		7	7	Min hî	Minimum total material handling costs	DEA/AHP methodology with the VisFactory tool
Jithavech and Krishnan (2010)	Risk-based stochastic FLP	7		7	7	Min R	Minimum maximum increased cost associated with risky flow intensities	Simulation-based approach and GA
Pillai et al. (2011)	Dynamic plant layout problem	7		7	7	Min hî	Minimum total material handling and relocation costs	SA-based meta-heuristic
Drira et al. (2013)	Fuzzy dynamic layout problem	7		7	7	The tr	The minimum sum of transportation costs	Fuzzy evolutionary algorithm
Izadinia et al. (2014)	Robust multi-floor layout problem	7		7	7	Mii hî	Minimum total material handling costs	MILP approach with linearization technique
Neghabi et al. (2014)	Robust multi-row FLP	7		7	7		Minimum total cost	A two-phase adaptive and iterative algorithm
Nematian (2014)	Fuzzy robust single-row facility layout problem	7		7	7	2	Minimum cost of transmitting all flow between departments	Fuzzy-stochastic chance-constrained programming based proposed exact algorithm
Salmani et al. (2015)	Robust multi-row FLP	7		7	7	2	Maximum number of adjacencies and minimum total layout area	MIP model
Vitayasak and Pongcharoen (2015b)	Robust machine layout design problem	7		2	7	Mii hi	Minimum total material handling distance	GA
Vitayasak and Pongcharoen (2015a)	Non-identical machine layout problem	7		7	7	Mii hi	Minimum total material handling costs	GA
Zhao and Wallace (2015)	Single-product capacitated distributed layout problem	2		>	2	Min m	Minimum total expected material handling cost	A greedy heuristic algorithm

Table 1 Summary of the literature review on robust facility layout design problems under uncertainty

Article	Problem type	Proble	Problem characteristics	istics					Objectives	Solution methodology
		Type .	Type of area	Uncertainty	ainty	Uncertain	Uncertain components			
		Equal	Unequal	Fuzzy	Stochastic	Demand	Breakdown (	Other		
Celik et al. (2016)	Stochastic distributed FLP	7			2	7			Minimum total expected flow distance	Dynamic programming-based heuristic procedure
Vitayasak and Pongcharoen (2016)	Robust machine layout problem	7			2	7	7		Minimum total material handling distance	Genetic Algorithm (GA)
Fazlelahi et al. (2016)	Robust dynamic single FLP	7			7	7			Minimum total cost	Permutation-based GA and robust optimization techniques
Zha et al. (2017)	Dynamic FLP		7	7		7			Minimum total material handling costs	Position-based flexible particle swarm optimization algorithm
Moslemipour et al. (2017)	Stochastic dynamic FLP	7			7	7			Minimum total material handling costs	QAP-based mathematical programming model
Morinaga et al. (2017)	Robust FLP	7			7		-	7	Minimum makespan for the obtained layouts	Conventional FLP method considering temporal efficiency
Moslemipour et al. (2018)	Stochastic dynamic facility layout problem	7			2	7			Minimum total cost	Hybrid AC-CS-SA algorithm
Peng et al. (2018)	Stochastic dynamic FLP	7			7	7			Minimum total material handling and rearrangement costs	Improved GA-based robust facility layout approach and Monte Carlo simulation method
Xiao et al. (2019)	Unequal-area dynamic FLP		7		7	7			Minimum total cost of wasted area	Robust optimization and an improved PSO algorithm
Vitayasak et al. (2019)	Robust machine layout problem	7			7	7	7		Minimum material flow distance	GA
Targhi et al. (2019)	Three-dimensional and multi-floor FLP		7		7		-	7	Minimum distances between departments	Mathematical programming approach
Hunagund et al. (2020)	Robust unequal area dynamic FLP		7		7	7	-	7	Minimum total material handling costs	SA algorithm
Khajemahalle et al. (2021)	Robust dynamic FLP	7			7	7	-	7	Total material handling and rearrangement costs	Hybrid algorithm based on nested partitions and the SA algorithm
Pourvaziri et al. (2022)	Robust FLD for flexible manufacturing		7		7	7			Minimum total material handling costs	DOE and a hybrid Genetic-Tabu search metaheuristic algorithm
Gölcük et al. (2022)	MADM for the facility layout evaluation		7	7			_	2	Total closeness rating score	A hybrid MADM model based on IT2F-FUCOM, ARCs, and MARCOS methods
Esmikhani et al. (2022)	Fuzzy robust FLP	7		7		2			Minimum total material handling costs	MPS and MNSGA-II

Table 1 (continued)

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Article	Problem type	Problem characteristics	teristics			Objectives		Solution methodology
		Type of area	Uncertainty	Uncert	Uncertain components			
		Equal Unequal		hastic Demar	Fuzzy Stochastic Demand Breakdown Other	Other		
Subulan et al. (2023)	Unequal-area capability-based machine layout	7				Minimize the total cost of material handling operations	cost of operations	Heuristic decomposition-based iterative mathematical programming approach
The present study	Robust CBDL problem	7	7 7	7	7	Minimum total product flow rate	luct flow	Hybrid chance-constrained program and an interactive fuzzy resolution method

unavailability) is stated as a discrete random variable with a known probability distribution, because demand quantities and the amounts of part flow among machines were generally defined as either fuzzy or stochastic parameters in many studies in the existing literature (Jithavech and Krishnan 2010; Zhao and Wallace 2015; Vitayasak and Pongcharoen 2015; Celik et al. 2016; Vitayasak et al. 2017; Xiao et al. 2019; Zha et al. 2017, 2020; Khajemahalle et al. 2021; Gölcük et al. 2022; Pourvaziri et al. 2022). On the other hand, since the machine breakdown is a random event as well as a risk issue, it was generally defined as a random variable with known probability distributions by the previous research on the facility layout design problems (Vitayasak and Pongcharoen 2016; Amri et al. 2016; Vitayasak et al. 2019). Since the proposed robust CBDL problem contains different types of uncertain variables together (i.e., fuzzy demand quantities and therefore, part flows and random machine breakdown), we proposed a hybrid solution approach based on the integration of chance-constrained stochastic programming (i.e., which is a well-known technique in probability theory) and an interactive fuzzy resolution method of Jimenez (2007) (i.e., which is also a well-known fuzzy mathematical programming approach in credibility and fuzzy set theory). To the best of our knowledge, there is no similar study in the literature on such a robust CBDL approach under mixed fuzzy and stochastic uncertainties. It should also be emphasized here that if there was an appropriate hybrid uncertain variable that contains both fuzziness and randomness inherently, we could also utilize the hybrid uncertainty and chance measure proposed by Liu (2007).

## 3 Description and mathematical formulation of the robust biased CBDL problem

The unbiased CBDL problem that doesn't take into account the demand and process flow information was first introduced in (Baykasoğlu 2003; Baykasoğlu and Göçken 2010). Actually, the main aim of the unbiased CBDL is to minimize the total distances from every unoccupied location by a RE to the location where RE is available. In other words, each processing capability can be accessed from every location of the factory layout with a minimum total distance. Afterward, Baykasoğlu and Subulan (2020) extended the UBCB-DL by incorporating the demand and process flow information of several parts and introduced a new problem, namely the biased CBDL design. Unlike the unbiased CBDL design problem, the biased version of this problem aims to minimize both the total part flow rates and distances from the unoccupied locations by REs to the occupied ones. In other words, in addition to minimize the

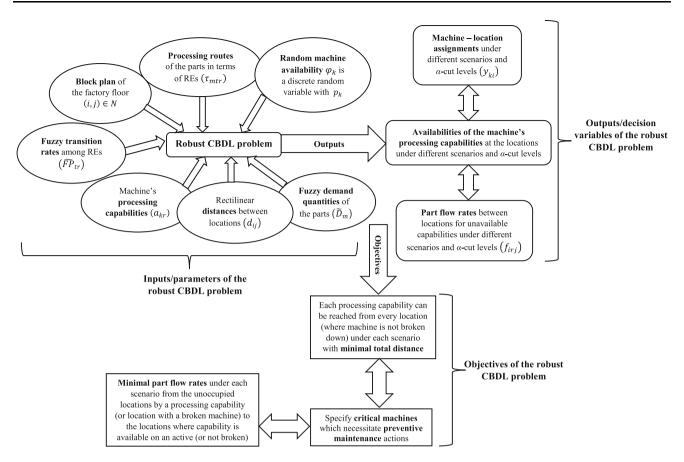


Fig. 1 Summary of the inputs/outputs and objectives of the proposed robust biased CBDL problem under a mixed fuzzy-stochastic environment

total distance covered, it is also targeted to achieve minimum part flow rates from the unoccupied locations by a machining capability to the locations where this capability is available. Therefore, all of the machining capabilities in terms of the REs can be distributed over the facility floor by considering the transition rates between different REs. On the other hand, these transition rates that are computed by the demand quantities and processing routes of the parts were assumed to take deterministic values in all of the previous studies on the capability-based layout design approaches (Baykasoğlu and Subulan 2020; Baykasoğlu et al. 2022; Subulan et al. 2023). However, demand quantities may include some sort of ambiguity, and therefore, uncertainty in these transition rates (or part flows) should be considered to provide a robust distributed layout design. Furthermore, random machine breakdowns are also crucial in designing robust layouts. For all of these reasons, this section introduces a new robust biased CBDL problem. To do this, the original MILP model formulation of the deterministic biased CBDL problem, which was formerly developed by Baykasoğlu and Subulan (2020), is first modified by incorporating probabilistic machine unavailability (or breakdowns) and fuzzy transition rates that show the dominant part flow patterns among different machining

capabilities (or REs). To this end, the inputs, outputs, and objectives of the proposed robust CBDL problem are summarized in Fig. 1. The modified MIP model with the probabilistic and fuzzy constraints is formulated in Eqs. (1)–(10) by using the mathematical nomenclature that is given in Table 2. The description of mathematical notation including model parameters and decision variables is also demonstrated in Fig. 1.

As shown in Fig. 1 and the objective function in Eq. (1), it is targeted to minimize the total part flow rates as well as the total distance from the unoccupied locations by the processing capabilities (or the locations with a broken machine) to the locations where these capabilities are available on active (or not broken) machines. In detail, parts wherever on the factory floor can access all of the required processing capabilities (REs) from their existing location within a minimum distance and material flow. Due to the unavailable REs and the broken machines, total rectilinear distances and part flow rates between different locations are considered simultaneously in Eq. (1).

Thus, all of the REs can be distributed appropriately over the facility floor by considering the fuzzy part flow (or transition) rates among different REs and the machine unavailability risks (or machinery breakdown). Indeed,

Table 2	Mathematical	nomenclature	for the	proposed	robust	CBDL problem	
---------	--------------	--------------	---------	----------	--------	--------------	--

Indices and Sets	
$(i,j) \in N$	Set of locations in the distributed layout
$k \in K$	Set of machines
$r \in R$	Set of Resource Elements (REs)
$m \in M$	Set of manufactured parts
Q	Set of parts that have transitions from RE $t$ to $r$ in their processing routes
Parameters	
$d_{ij}$	Rectilinear distance between locations <i>i</i> and <i>j</i>
$a_{kr}$	1, if machine $k$ has processing capabilities including the RE $r$ ; 0 otherwise
$\tilde{D_m}$	Fuzzy demand quantity for part m
$ au_{mtr}$	Number of transitions from RE $t$ to $r$ in the process route of part $m$
$\widetilde{FP}_{tr}$	Fuzzy transition rates/flow patterns between the RE $t$ and $r$ in the process routes of all parts
$arphi_k$	Discrete random variable for the availability of machine k
$p_k$	Probability for the availability of machine k
$\theta$	Satisfaction probability of the chance-constraint set in Eq. (5)
β	Satisfaction probability of the chance-constraint set in Eq. (6)
S	Scenario or sample size in the chance-constrained stochastic program
Μ	A huge positive number
Decision variables	
<i>Y</i> <sub>ki</sub>	1, if machine $k$ is assigned to location $i$ ; 0 otherwise
Z <sub>irj</sub>	1, if i is the closest location to location j for the unavailable RE $r$
$f_{irj}$	Part flow rate from location $i$ to $j$ for the unavailable RE $r$

there will be part flows from an unoccupied location by a RE to any occupied one in order to achieve the required processing capability. It should also be noted here that if the assigned machine to this location is broken down, part flows should be realized by choosing another occupied location with an active machine including that RE. By the way, there may also be part flows from an occupied location by a RE to another occupied one because of the machine breakdown. In addition to minimize the total part flow rates and total distances as formulated in Eq. (1), the proposed robust BCB-DL approach also aims to predetermine the critical machines which require preventive maintenance actions. Actually, these critical machines can be obtained from the stochastic optimization results, which display the scenarios with unavailable or broken machines.

$$\text{Minimize} \sum_{i \in N} \sum_{j \in N \setminus \{i\}} \sum_{r \in R} (f_{irj} + Z_{irj}) \cdot d_{ij} \tag{1}$$

Subject to:

$$\sum_{i\in\mathbb{N}} y_{ki} = 1 \quad \forall k \in K \tag{2}$$

$$\sum_{k \in K} y_{ki} = 1 \quad \forall i \in N \tag{3}$$

$$\sum_{\substack{j \in N \setminus \{i\} \\ \forall i \in N \setminus \{j\}, \forall r \in R}} Z_{irj} \leq (|N| - 1). \left(1 - \sum_{j \in N \setminus \{i\}} Z_{jri}\right)$$
(4)

$$\Pr\left\{\sum_{j\in N\setminus\{i\}} Z_{jri} \ge 1 - \sum_{k\in K} a_{kr} \cdot y_{ki} \cdot \varphi_k\right\} \ge \theta \quad \forall i \in N, \forall r \in R$$
(5)

$$\Pr\left\{\sum_{k\in K}\sum_{t\in R\setminus\{r\}}\widetilde{FP}_{tr}\cdot a_{kt}\cdot y_{ki}\cdot \varphi_k - \sum_{j\in N\setminus\{i\}}f_{irj}\leq M\cdot\left(1-z_{jri}\right)\right\}$$
  
$$\geq\beta\;\forall i\in N,\forall r\in R$$

(6)

$$f_{irj} \le M \cdot Z_{jri} \quad \forall i \in N, \forall j \in N \setminus \{i\}, \forall r \in R$$
(7)

$$f_{irj} \ge 0$$
 and continuous  $\forall i \in N, \forall j \in N, \forall r \in R$  (8)

$$y_{ki}, z_{ijr} \in \{0, 1\} \quad \forall i \in N, \forall j \in N, \forall k \in K, \forall r \in R$$
(9)

$$\varphi_k$$
 is a discrete random variable with probability of  $p_k$   
 $\forall k \in K$ 

(10)

Let us explain the constraints of the proposed fuzzystochastic MIP model. According to the constraint set in Eq. (2), each machine can only be assigned to one location on the factory floor. Additionally, when a machine is assigned to a location, then all of its processing capabilities/REs are also occupied in that location. This study also assumed that each location on the facility floor has an equal size, which is able to locate any machine. Constraint set in Eq. (3) guarantees that only one machine can be assigned to each location on the factory floor. Actually, these assignment constraints in Eqs. (2)–(3) aim to choose the optimal locations of the machines. According to the constraint set in Eq. (4), if location *i* is not occupied by the RE r, whereas this processing capability is available at the nearest location *i*, a part flow from location *i* to *j* will be carried out to achieve that RE. In this situation, the binary variable, i.e., *z<sub>iri</sub>*, takes the value of "1," which means that there is a part flow from location i to j to access RE r. Moreover, this constraint also maintains that the part flows can be realized to this occupied location *j* by the RE *r* from the maximum number of INI-1 unoccupied locations on the facility floor. In fact, |N|-1 is utilized in this formula to avoid the usage of a positive Big M value (Baykasoğlu and Subulan 2020). The chance constraint set in Eq. (5) ensures that if a machine k, which doesn't include the RE r is assigned to location *i*, then this location needs to achieve RE r with minimum distance. In detail, a part flow is needed from location i to the nearest location j where RE ris available. It should be noted here that the probabilistic breakdown of machine k has been already considered by this constraint with the stochastic right-hand side. In other words, the random machine unavailability is formulated by this chance constraint set because the random machine breakdowns cannot be known with certainty in real-life dynamic manufacturing environments but may have known probability distributions. Thus, it is assumed that in a longterm planning horizon, machine k may be in active status with a pre-specified probability of  $p_k$ , and therefore, it may also be in inactive or broken status with an unavailability probability of  $(1 - p_k)$ . If machine k that is assigned to location *i* is broken down, then all the processing capabilities will disappear automatically from that location i. For that reason, a robust distributed layout design that considers the machine unavailability risk or breakdowns is generally desired by the facility designers. Thus, the availability of machine k, i.e.,  $\varphi_k$ , is defined as a discrete random variable with known probability. If machine kbecomes an active statue (or available), then  $\varphi_k$  will take the value of "1" with the probability of  $p_k$ . Otherwise, if it becomes inactive status, then this random variable will take the value of "0" with the probability of  $(1 - p_k)$ . This also means that  $p_k$  corresponds to the machine reliability in the long-term planning horizon. With the help of the random machine unavailability risk formulation in this chance constraint set in Eq. (5), although the machine k which is assigned to location *i* includes the RE *r*, it may not be in active status because of the machinery breakdown. In this case, a part flow must be realized again from location *i* to the nearest location *j* for reaching that RE *r*. By using the chance constraint set in Eq. (6), part flow rates among various facility locations can be computed for different REs. Indeed, these flow rates can be calculated as a result of the generated distributed layout design. In other words, their values mainly depend on the machine location assignments. For that reason, if the RE r is not included by machine k that is assigned to location i, there will be part flows from that location *i* to the nearest location *j* including that RE r. Thus, the number of part flows should be greater than the total transition rates between the available REs  $(t \in R)$  and unavailable REs r  $(r \in R \text{ and } r \neq t)$  at this location *i*. In Eq. (6), the probabilistic machine unavailability risks are again considered since the machine k with RE t may be broken down, and therefore, this location also needs to reach the RE t as well as RE r. In this situation, a part flow from location i to its nearest location j is needed for both cases to reach the RE r. However, it should be emphasized here that there is an inaccessibility risk of that RE r from the unoccupied location i because of this chance-constrained stochastic programming formulation. In fact, these probabilistic constraint sets in Eqs. (5)-(6) may not be satisfied 100% of the time since the machine availabilities are defined as discrete random variables with known probabilities. Instead, it is targeted to find a robust CBDL that will satisfy these probabilistic constraints under a pre-specified percentage, i.e., 90 or 95 percent of the time. In other words, the generated distributed layout alternatives may be feasible for 95% or 90% of all instances for the random machine breakdowns.

It should also be noted here that if one or more machines are broken down in any scenario, they may cause a violation of the chance constraint sets in Eqs. (5)–(6). In this case, one or both of these probabilistic constraints will be unsatisfied in that scenario. Therefore, the unsatisfied scenarios including the machinery breakdowns will show critical machines. Thus, we cannot distribute the machines' processing capabilities (or REs) appropriately over the facility floor in case of critical machines are broken down. In other words, there will be no feasible distributed layout alternative for these unsatisfied scenarios. On the contrary, even if some broken machines are available, there may be no unsatisfied scenario in stochastic optimization results. In this case, there will be no critical machines, and therefore, machine unavailability risks may be insignificant. Actually, we may encounter this situation when all of the REs or processing capabilities are covered by many alternative machines. Thus, when a machine including the required RE is broken down, alternative machines can come into play. On the other hand, if all of the alternative machines for a

RE are broken down, the probabilistic chance constraints are not satisfied in that scenario. In a nutshell, capability overlaps shown in the binary machine-capability matrix  $(a_{kr})$  play a significant role in stochastic optimization results, which help us to determine these critical machines. In addition to the random machine breakdowns, we may not know the dominant flow patterns between different REs with certainty, because the demand quantities of the manufactured parts may have some sort of ambiguity in many real-life manufacturing environments. Therefore, transition frequencies or flow rates among different REs, i.e.,  $FP_{tr}$ are also considered as uncertain layout parameters in Eq. (6) and represented by triangular fuzzy numbers. In order to calculate fuzzy flow rates and then construct the transition rate matrix, we formulated Eq. (11), which mainly uses the fuzzy demand quantities and the prespecified deterministic processing routes of parts. According to this formula, the number of transitions among various REs should be first obtained from the processing routes of all the manufactured parts. Afterward, the fuzzy transition rate matrix can be computed as in Eq. (11) by using the fuzzy division operator. On the other hand, it should be highlighted here that this transition rate matrix may not be always constant and change according to the dynamic demand fluctuations in the future. Moreover, additional processing operations (or route changes) of new parts or discontinued production of some outdated parts may also affect this transition rate matrix. For all of these reasons, the construction of a fuzzy transition rate matrix may be more convenient to generate robust layout design dynamic and uncertain manufacturing options in environments.

$$\widetilde{FP}_{tr} = \frac{\sum_{m \in Q} \tilde{D}_m \cdot \tau_{mtr}}{\sum_{m \in M} \sum_{t \in R} \sum_{r \in R \setminus \{t\}} \tilde{D}_m \cdot \tau_{mtr}} \quad \forall (t, r) \in R \quad (11)$$

Finally, Eq. (7) ensures that if location j is not the closest point to location i for reaching the RE r, there will be no part flows between these locations. To satisfy this statement, a big M value is utilized to avoid the redundant part flows over the facility floor. In Eq. (8), the part flow rates computed by Eq. (6) are defined as continuous variables. Furthermore, it should be noted here that the total amount of part flow rates should be equal to "1." Equation (9) ensures the binary integrality of the variables for the machine location assignments and part flow requirements for the unavailable REs between different locations of the factory floor. Lastly, in Eq. (10), machine availability is defined as a discrete random variable with known probability.

### 4 The proposed hybrid solution approach

This section presents a hybrid solution approach based on a chance-constrained stochastic program with a fuzzy resolution approach. Actually, we employed the chance-constrained stochastic programming approach of Charnes and Cooper (1959) simultaneously with a well-known interactive fuzzy resolution method of Jimenez et al. (2007) to transform the proposed fuzzy-stochastic optimization model into its crisp equivalent form. Indeed, since the probabilistic constraint set in Eq. (6) incorporates both fuzzy and stochastic parameters, the usage of such a hybrid solution approach is necessary. Baykasoğlu and Subulan (2019) also employed a similar hybrid approach while dealing with a fuzzy-stochastic intermodal fleet management problem. In a similar way, the chance-constrained stochastic programming approach is first utilized in this paper to handle the probabilistic constraint set in Eq. (5) in which the discrete random variables are defined only for the uncertain machine breakdowns with known probability values. In fact, we considered two different probability distribution functions for the random machine breakdowns to analyze the effects of these distributions on the stochastic optimization results: (i) Basic discrete random variable with known probabilities and (ii) Binomially distributed random machine breakdowns with one trial only (n = 1) and the success probabilities of  $p_k$ . Actually, this distribution case also corresponds to Bernoulli random variables because of using only one trial. This means that any machine can be broken down only once in the planning period or the facility designer desires to take only one sample from the manufacturing environment for random machine unavailability. If the number of trials is increased, the facility designer needs to collect more data related to machine failures and breakdown statistics. When the basic discrete random variables with known probabilities are defined for the probabilistic machine breakdowns, the chance constraint set in Eq. (5) can be converted into its deterministic equivalent nonlinear form as in Eq. (14) by using critical values of these random machine breakdowns. To do this, these critical values, i.e.,  $\psi_{ir}$ , can be computed by integrating the probability density function in Eq. (12)and inverse cumulative distribution function (see in Eq. 13) of this discrete random variable, i.e.,  $\varphi_k$  (Bisschop and Roelofs 2006; Gupta et al. 2003; Subulan 2020). Let us define the cumulative distribution function of this basic discrete random variable, i.e.,  $F(\psi_{ir}) = \zeta_{ir} \forall i \in N, \forall r \in R$ , and then, the deterministic approximation of this probabilistic constraint in Eq. (5) can be formulated as in Eqs. (12)–(14) by using its inverse cumulative (or quantile) function, i.e.,  $F^{-1}(\zeta_{ir}|p_k)$ .

$$f_x(\varphi_k) = \begin{cases} 1 & \text{if machine } k \text{ is in active statue with probability of } p_k \\ 0 & \text{if machine } k \text{ is broken down with probability of } (1-p_k) \end{cases} \forall k \in K$$
(12)

1

$$\psi_{ir} = F^{-1}(\zeta_{ir}|p_k) = \begin{cases} 1 & 0 < \varphi_k \le 1\\ 0 & \varphi_k \le 0 \end{cases} \quad \forall k \in K$$

$$(13)$$

$$1 - \sum_{j \in N \setminus \{i\}} Z_{jri} \le \sum_{k \in K} a_{kr} \cdot y_{ki} \cdot \psi_{ir} \ \forall i \in N, \forall r \in R$$
(14)

On the other hand, when the binomially distributed random variables are defined for the probabilistic machine availabilities, these critical values, i.e.,  $\psi_{ir}$ , can be determined as in Eq. (15) by using the inverse cumulative function of the binomial distribution.

$$\psi_{ir} = F^{-1}(\zeta_{ir}|w;n,p_k) = \sum_{q=1}^{w} \binom{n}{q} \cdot (p_k)^q \cdot (1-p_k)^{n-q} \ \forall i$$
  

$$\in N, \forall r \in R$$
(15)

where w represents the number of successes (or active statue of machine k with a probability of  $p_k$ ) in an independent Bernoulli trial (n = 1). Similarly, the chance constraint set in Eq. (6) should also be transformed into its deterministic equivalent non-linear form. However, this chance constraint set involves not only probabilistic machine availabilities  $(\varphi_k)$  but also includes a fuzzy parameter, i.e.,  $\overline{FP}_{tr}$ , that corresponds to uncertain transition frequencies (or part flow rates) between different machining capabilities/REs. For that reason, the  $\alpha$ -parametric approach of Jimenez et al. (2007) is first applied to transform this fuzzy constraint into its crisp equivalent form. Afterward, the chance-constraint stochastic programming approach is implemented again to deal with the random machine breakdowns. The fuzzy technological coefficient, i.e.,  $\widetilde{FP}_{tr}$ , which is located on the left-hand side of this probabilistic constraint in Eq. (6) is represented by a triangular fuzzy number, i.e.,  $(FP_{tr}^{p}, FP_{tr}^{m}, FP_{tr}^{o})$ . In fact, the left, core, and right margins of this triangular fuzzy number display the pessimistic, most likely, and optimistic values of the uncertain part flow rates, respectively. The expected interval of this fuzzy parameter can be computed as in Eq. (16). In addition,  $\alpha$ -cut based fuzzy ranking procedure of Jimenez (1996) is employed to carry out the fuzzy to crisp transformation process. After completing this transformation process, the chance constraint set in Eq. (6) can be reformulated as in Eq. (17):

$$EI\left(\widetilde{FP}_{tr}\right) = \left[\frac{1}{2} \cdot \left(FP_{tr}^{p} + FP_{tr}^{m}\right), \frac{1}{2} \cdot \left(FP_{tr}^{m} + FP_{tr}^{o}\right)\right]$$
(16)

$$\Pr\left\{\sum_{k}\sum_{t\in\mathbb{R}\setminus\{r\}}\left(\alpha\cdot\frac{FP_{tr}^{m}+FP_{tr}^{o}}{2}+(1-\alpha)\cdot\frac{FP_{tr}^{\rho}+FP_{tr}^{m}}{2}\right)\right.$$
$$\left.\cdot a_{kt}\cdot y_{ki}\cdot \varphi_{k}-\sum_{j\in\mathbb{N}\setminus\{i\}}f_{irj}\leq M\cdot\left(1-z_{jri}\right)\right\}\geq\beta\;\forall i\in\mathbb{N},\forall r\in\mathbb{R}$$
$$(17)$$

where  $\alpha$  is a parameter that demonstrates the feasibility degree of the constraint set in Eq. (17). When the value of this parameter is equal to '0,' it corresponds to an unacceptable or the most risky solution. On the other hand, one can achieve a completely acceptable or risk-free solution when its value is equal to '1.' Through this parameter whose value ranged from '0' to '1,' various layout design alternatives can be produced under different uncertainty levels. In other words, many  $\alpha$ -acceptable layout design options can be produced by taking into account the conflict between the satisfaction degree of layout objectives and the fulfillment degree of this fuzzy constraint. After performing the fuzzy to crisp transformation procedure, the chance constraint set in Eq. (17) should also be converted into its deterministic equivalent nonlinear form as in Eq. (14). Similar to the previous formulations in Eqs. (12)-(15), critical values of the random machine availabilities, i.e.,  $\psi_{ir}$ in Eq. (18) can be calculated by using its inverse cumulative distribution functions, i.e.,  $F^{-1}(\zeta_{ir}|p_k)$ or  $F^{-1}(\zeta_{ir}|w;n,p_k)$ . A summary of the proposed hybrid solution approach is also depicted in Fig. 2.

$$\sum_{j \in N \setminus \{i\}} f_{irj} + M \cdot (1 - z_{jri})$$

$$\geq \sum_{k} \sum_{r \in R \setminus \{r\}} \left( \alpha \cdot \frac{FP_{tr}^{\rho} + FP_{tr}^{m}}{2} + (1 - \alpha) \cdot \frac{FP_{tr}^{m} + FP_{tr}^{o}}{2} \right) \cdot a_{kt} \cdot y_{ki} \cdot \psi_{ir}$$

$$\forall i \in N, \forall r \in R$$
(18)

Unfortunately, it should be emphasized here that application of the aforementioned fuzzy random to crisp transformation may cause a highly nonlinear deterministic equivalent mathematical model. Moreover, the solution of this kind of mathematical program via the standard MIP solvers of any optimization software is a challenging task, because several auxiliary variables and additional constraints are needed to carry out these transformation

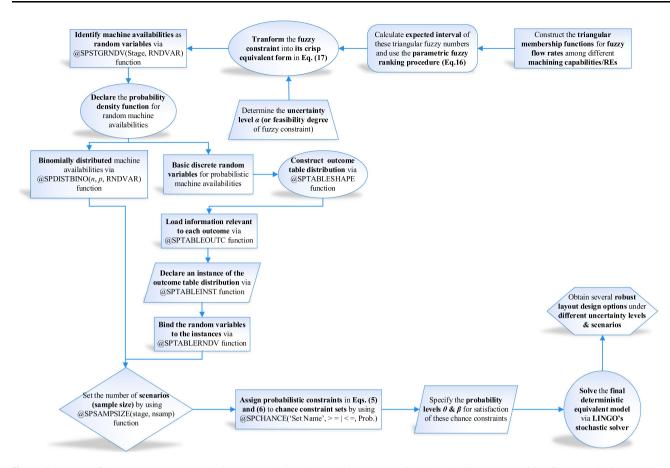


Fig. 2 Summary of the proposed hybrid solution approach based on a chance-constrained stochastic program with a fuzzy resolution approach

operations. As a result, the required computational effort for the solution of this kind of mathematical program will increase significantly concerning the problem dimension. To overcome this problematic issue, we use some special commands and functions like @SPSTGRNDV, @SPDIST-BINO, @SPTABLESHAPE, @SPTABLEOUTC, @SPTABLEINST, @SPTABLERNDV, @SPSAMPSIZE, @SPCHANCE, etc. of LINGO 19.0 optimization software to construct the chance constraint sets in Eqs. (5)–(6) and used its powerful stochastic programming solver to solve the present problem (LINDO Systems Inc. 2023). Actually, the stochastic solver of LINGO 19.0, which can apply Benders Decomposition and Genetic Algorithms, is used to achieve high-quality robust solutions for the current fuzzy-stochastic biased CB-DL problem.

## 5 Computational study with comparative analysis

In this section, a numerical example is presented to illustrate the proposed robust biased CBDL problem and analyze the effects of different machine capability overlap cases on the fuzzy-stochastic optimization results.

#### 5.1 Data description for an illustrative example

The numerical example is composed of 6 machines and 6 locations in a  $2 \times 3$  block plan, and the machine-processing capability information related to the REs is shown in Table 3. Each machine has a set of processing capabilities which are described in terms of REs and total 6-REs are available. The processing capability information of machines given in Table 3 is also depicted in Fig. 3c, which corresponds to the medium capability overlap case. Based on this case, five different machine-RE matrices are derived for "*no*," "*low*," "*high*," "*very high*," and "*fully overlap*" cases (see Fig. 3a-f) to explore the impact of different machine capability overlap cases on the optimization results.

It is also intended to investigate the effects of the random machine breakdowns on fuzzy-stochastic optimization

Table 3 Processing capabilities of machines in terms of the REs

Machine-1: RE1, RE2, RE3	Machine-4: RE3, RE4, RE6
Machine-2: RE1, RE2, RE5	Machine-5: RE4, RE5, RE6
Machine-3: RE1, RE3, RE4	Machine-6: RE2, RE5, RE6

			R	Es							RI	Es							RI	Es		
Machines		(N	0 0	verla	ıp)		Mach	ines		(Lo	w o	verl	ap)			Machines	[]	Med	ium	ove	erlap	)
(a)	1	2	3	4	5	6	(b)	) [	1	2	3	4	5	6		(c)	1	2	3	4	5	6
1	*						1		*	*						1	*	*	*			
2		*					2		*	*						2	*	*			*	
3			*				3				*	*				3	*		*	*		
4				*			4				*	*				4			*	*		*
5					*		5						*	*		5				*	*	*
6						*	6						*	*		6		*			*	*
			R	Es							R	Ea			Г				ום			
Machines		<i>(</i> Ц;	gh c		lon)		Mash		a					(		3.4 1.		Œ	RI		1 \	
		<u>`</u>	ř –	r –	<b>^</b>		Mach		(1)				erla			Machines		(Fu			<b>*</b> /	
(d)	1	2	3	4	5	6	(e)	)	1	2	3	4	5	6		(f)	1	2	3	4	5	6
1	*	*	*		*		1		*	*	*		*	*		1	*	*	*	*	*	*
2	*	*		*	*		2		*	*	*	*	*			2	*	*	*	*	*	*
3	*		*	*		*	3		*	*	*	*		*		3	*	*	*	*	*	*
4	*		*	*		*	4		*		*	*	*	*		4	*	*	*	*	*	*
5		*		*	*	*	5		*	*		*	*	*		5	*	*	*	*	*	*
6		*	*		*	*	6			*	*	*	*	*		6	*	*	*	*	*	*

### \* RE is available

Fig. 3 a No, b low, c medium, d high, e very high, and f fully machine capability overlap cases

**Table 4** Fuzzy demandquantities and processing routesof the manufactured parts

Parts	Demands (in units)	Process flow information in terms of REs (Processing routes)
1	(5, 10, 15)	RE1-RE3-RE4-RE6-RE1
2	(20, 30, 40)	RE2-RE4-RE5-RE3
3	(5, 15, 20)	RE1-RE2-RE5-RE4-RE3-RE6
4	(10, 20, 30)	RE1-RE4-RE2-RE1-RE5-RE2-RE3
5	(40, 50, 60)	RE1-RE6-RE3-RE2-RE6-RE4-RE1
6	(5, 10, 20)	RE3-RE1-RE6-RE5-RE1
7	(50, 70, 80)	RE3-RE5-RE6-RE2

\*RE is available

Table 5 Fuzzy transition rates (or uncertain part flow patterns) between different REs

RE1	RE2	RE3	RE4	RE5	RE6
(0,0,0)	(0.01,0.016,0.016)	(0.01,0.011,0.012)	(0.019,0.021,0.024)	(0.019,0.021,0.024)	(0.128,0.131,0.14)
(0.019,0.02,0.024)	(0,0,0)	(0.019,0.021,0.024)	(0.032,0.032,0.032)	(0.01,0.016,0.016)	(0.05,0.053,0.064)
(0.01,0.01,0.016)	(0.05,0.053,0.064)	(0,0,0)	(0.01,0.011,0.012)	(0.07,0.075,0.08)	(0.01,0.016,0.016)
(0.05,0.053,0.064)	(0.019,0.021,0.024)	(0.01,0.016,0.016)	(0,0,0)	(0.032,0.032,0.032)	(0.01,0.011,0.012)
(0.01,0.011,0.016)	(0.019,0.021,0.024)	(0.032,0.032,0.032)	(0.01,0.016,0.016)	(0,0,0)	(0.07,0.075,0.08)
(0.01,0.011,0.012)	(0.07,0.075,0.08)	(0.05,0.053,0.064)	(0.05,0.053,0.064)	(0.01,0.011,0.016)	(0,0,0)

results in case of different machine capability overlaps. To do this, different probability values (85% and 90%) for the random machine availabilities are also tested for each machine capability overlap case. Indeed, each machine will become active status with a pre-specified probability of  $p_k$ . Otherwise, any machine may break down, and therefore, it

will be inactive status with the probability of  $(1 - p_k)$ . Actually, this probability information for the machine availabilities is one of the most important layout parameters for both discrete random variables and binomial distribution. Furthermore, fuzzy demand data with the

Locations	Locat	tions				
	1	2	3	4	5	6
1	0	1	2	1	2	3
2	1	0	1	2	1	2
3	2	1	0	3	2	1
4	1	2	3	0	1	2
5	2	1	2	1	0	1
6	3	2	1	2	1	0

Table 6 Unit part flow costs among fixed locations

manufacturing routes (in terms of REs) of 7 different parts are displayed in Table 4.

Based on this process flow information and fuzzy demand data, the fuzzy transition rate matrix (or fuzzy part flow rates among different REs) is computed by using Eq. (11) and presented in Table 5. It should be noted here that sum of the part flow rates in this table is also equal to a fuzzy number, i.e., (0.918, 0.9996, 1.1233) which covers the value of "1" as an expected result. Finally, unit part flow (or material handling) costs are computed based on the symmetric rectilinear distances between different locations on the factory floor and given in Table 6.

## 5.2 Comparative results for deterministic, fuzzy, stochastic, and fuzzy-stochastic cases

When the proposed fuzzy-stochastic programming model is run with these data through the stochastic solver of LINGO 19.0 optimization software on an Intel Corei7 2 GHz IBM PC, comparative results are provided for different capability overlap cases with distinct machine availabilities and satisfaction probabilities of the chance constraint sets as given in Tables 7, 8, 9. The details of the deterministic, fuzzy, stochastic, and fuzzy-stochastic optimization cases are first given in Table 7, where the sample size is set to 5, machine availabilities and the satisfaction probabilities for the chance constraint sets are equal to 90% and 80-90%, respectively. The optimization details such as the total number of random and integer variables, constraints, CPU time, total solver iterations, and extended solver steps are also reported in that table. It should be emphasized here that the expected values of triangular fuzzy numbers are calculated by using Eq. (19) while obtaining the deterministic and stochastic optimization results. It is also assumed that each machine will be active (or 100% machine availability) in the deterministic cases. This assumption is also valid for the fuzzy optimization cases, where  $(\alpha = 0 \text{ and } \alpha = 1)$  (Jimenez et al. 2007; Jimenez 1996).

$$EV\left(\widetilde{FP}_{tr}\right) = \frac{FP_{tr}^{p} + 2.FP_{tr}^{m} + FP_{tr}^{o}}{4} \ \forall (t,r) \in R$$
(19)

It should also be noted here that the probabilistic machine availabilities are defined as basic discrete random variables with known probabilities in the stochastic optimization case-1. On the other hand, these machine availabilities are assumed to fit a binomial distribution in the stochastic optimization case-2 to compare the results of different probability distributions. In the fuzzy-stochastic optimization cases, these different probability distributions for the random machine availabilities or breakdowns are also taken into account in addition to the fuzzy part flow rates. In the stochastic and fuzzy-stochastic optimization cases, the satisfaction probabilities for the chance constraints in Eqs. (5)-(6) are set to 80% and 90%, respectively (see Table 7). According to the comparative analysis in Table 7, the layout objective, which consists of total distances and part flow rates decreases in all of the deterministic, fuzzy, stochastic, and fuzzy-stochastic cases when the machine capability overlap is increased. For instance, the total layout score of the no overlap case is significantly high (i.e., 51.41448) in deterministic optimization when compared to the objective value of the fully overlap case (i.e., 0). Thus, it is clear that the degree of machine capability overlap has a substantial effect on the layout objective. It is also obviously seen from Table 7 that the random machine unavailability has also a considerable impact on the layout objective. When the deterministic and stochastic optimization cases are compared, it can be inferred that the layout score will increase due to random machine breakdowns. It should also be highlighted here that a feasible solution cannot be found for no overlap cases in all of the stochastic and fuzzy-stochastic optimization cases due to the unavailability of critical machines. It is also clearly seen in Table 7 that binomially distributed random machine availability may cause larger values for the layout objectives. Furthermore, the deterministic equivalent form of the binomial distribution may cause a larger number of integer variables, but fortunately, the same number of total constraints. According to Table 7, there is no significant difference between the layout scores or objective values of the deterministic and fuzzy optimization cases.

Indeed, the layout scores of deterministic cases took values between the lowest ( $\alpha = 1$ ) and highest ( $\alpha = 0$ ) uncertainty levels of the fuzzy optimization cases. This means that the uncertainty in the part demands has not a crucial influence on the total layout score as much as the random machine breakdowns. Therefore, layout scores of the fuzzy optimization cases are relatively low as in the deterministic cases since the random machine breakdowns are not considered. However, the layout score will increase in the fuzzy optimization cases when a risk-free solution

		No overlap	Low overlap	Medium overlap	High overlap	Very high overlap	Fully overlap
Deterministic case	Overall layout score	51.41448	25.72375	20.05333	13.7145	7.010125	0
	CPU time (sec.)	8.59	1.33	15.96	7.61	16.12	0.09
	Total solver iterations	101,030	15,320	293,150	75,531	224,817	0
	Extended solver steps	680	247	11,746	5416	21,765	0
	Total integer variables	252					
	Total constraints	481					
Stochastic case-1 (discrete random variables with known probabilities)	Total random variables	6					
	Total integer variables (Deteq.)	258					
	Total constraints (Deteq.)	771					
	Overall layout score	Infeasible (N.A)	31.99182	24.20307	18.119	12.71015	7.01012
	# of unsatisfied scenarios for CCP1	5	1	1	1	1	1
	Actual probability for CCP1	0%	80%	80%	80%	80%	80%
	# of unsatisfied scenarios for CCP2	5	0	0	0	0	0
	Actual probability for CCP2	0%	100%	100%	100%	100%	100%
	CPU time (sec.)	1.38	1.28	5.63	6.89	4.82	1.32
	Total solver iterations	0	5782	83,376	93,312	43,281	6568
	Extended solver steps	0	14	971	1655	2809	99
Stochastic case-2 (Binomial distribution)	Total random variables	6					
	Total integer variables (Deteq.)	260					
	Total constraints (Deteq.)	771					
	Overall layout score	Infeasible (N.A)	38.25903	30.54395	23.6171	18.41923	14.0202
	# of unsatisfied scenarios for CCP1	5	1	1	1	1	1
	Actual probability for CCP1	0%	80%	80%	80%	80%	80%
	# of unsatisfied scenarios for CCP2	5	0	0	0	0	0
	Actual probability for CCP2	0%	100%	100%	100%	100%	100%
	CPU time (sec.)	1.61	2.99	11.42	23.12	13.76	4.76
	Total solver iterations	0	41,380	143,314	252,765	151,771	58,376
	Extended solver steps	0	19	518	1760	1760	933
Fuzzy case ( $\alpha = 0$ )	Overall layout score	51.3365	25.6386	19.9593	13.6331	6.9588	0
	CPU time (sec.)	8.77	0.96	9.74	8.86	5.35	0.009
	Total solver iterations	10,829	9954	155,883	59,457	32,755	0
	Extended solver steps	723	132	4090	3373	2970	0

#### Table 7 (continued)

		No overlap	Low overlap	Medium overlap	High overlap	Very high overlap	Fully overlap
Fuzzy case $(\alpha = 1)$	Overall layout score	51.49245	25.8089	20.14735	13.7959	7.06145	0
	CPU time (sec.)	7.28	1.08	5.02	17.14	19.49	0.09
	Total solver iterations	87,588	12,255	99,820	181,584	203,952	0
	Extended solver steps	503	192	3796	12,315	19,050	0
Fuzzy-stochastic case-1 ( $\alpha = 0$ )	Overall layout score	Infeasible (N.A)	31.8928	24.0983	18.0151	12.6214	6.9588
	# of unsatisfied scenarios for CCP1	5	1	1	1	1	1
	Actual probability for CCP1	0%	80%	80%	80%	80%	80%
	# of unsatisfied scenarios for CCP2	5	0	0	0	0	0
	Actual probability for CCP2	0%	100%	100%	100%	100%	100%
	CPU time (sec.)	1.43	1.32	4.0	8.2	13.53	1.29
	Total solver iterations	0	8505	53,461	115,247	154,565	6501
	Extended solver steps	0	39	263	1376	6722	214
Fuzzy-stochastic case-2 ( $\alpha = 0$ )	Overall layout score	Infeasible (N.A)	38.1482	30.4241	23.4899	18.2956	13.9176
	# of unsatisfied scenarios for CCP1	5	1	1	1	1	1
	Actual probability for CCP1	0%	80%	80%	80%	80%	80%
	# of unsatisfied scenarios for CCP2	5	0	0	0	0	0
	Actual probability for CCP2	0%	100%	100%	100%	100%	100%
	CPU time (sec.)	2.44	2.79	7.91	13.76	13.98	5.97
	Total solver iterations	0	34,099	108,925	147,976	160,771	75,157
	Extended solver steps	0	29	248	1061	1861	1234
Fuzzy-stochastic case-1 ( $\alpha = 1$ )	Overall layout score	Infeasible (N.A)	32.09085	24.30785	18.2229	12.7989	7.06145
	# of unsatisfied scenarios for CCP1	5	1	1	1	1	1
	Actual probability for CCP1	0%	80%	80%	80%	80%	80%
	# of unsatisfied scenarios for CCP2	5	0	0	0	0	0
	Actual probability for CCP2	0%	100%	100%	100%	100%	100%
	CPU time (sec.)	1.47	1.15	8.91	6.44	6.51	0.69
	Total solver iterations	0	6665	126,006	74,110	81,924	5247
	Extended solver steps	0	46	1350	1056	3227	149

#### Table 7 (continued)

		No overlap	Low overlap	Medium overlap	High overlap	Very high overlap	Fully overlap
Fuzzy-stochastic case-2 ( $\alpha = 1$ )	Overall layout score	Infeasible (N.A)	38.36985	30.6638	23.7443	18.54285	14.1229
	# of unsatisfied scenarios for CCP1	5	1	1	1	1	1
	Actual probability for CCP1	0%	80%	80%	80%	80%	80%
	# of unsatisfied scenarios for CCP2	5	0	0	0	0	0
	Actual probability for CCP2	0%	100%	100%	100%	100%	100%
	CPU time (sec.)	1.34	3.54	8.27	17.92	14.08	5.07
	Total solver iterations	0	42,150	104,781	191,261	136,713	43,373
	Extended solver steps	0	26	464	1180	1469	476

Sample size = 5; machine availabilities = 90%; probability for breakdowns = 10%; probabilities for the chance-constraint sets = 80% and 90% Deterministic equivalent form (Deteq.)

 $(\alpha = 1)$  is desired by the facility designer. If the most risky solution is provided under the highest uncertainty level, this layout score will also reduce ( $\alpha = 0$ ). Therefore, various layout design alternatives can be obtained by changing the value of these parameters.

Finally, when the results of fuzzy-stochastic optimization cases, which consider both fuzzy and stochastic layout design parameters, are evaluated, it can be concluded that the layout scores took relatively fewer values when compared to the stochastic optimization cases ( $\alpha = 0$ ). Nevertheless, the layout scores will take larger values than the stochastic optimization cases when the risk-free layout design is needed ( $\alpha = 1$ ). The number of unsatisfied scenarios is equal to "1" for the first chance constraint set in Eq. 5 (CCP1) in all the stochastic and fuzzy-stochastic optimization cases. Therefore, the actual satisfaction probability of the CCP1 is also equal to its target probability (i.e., 80%). On the other hand, since all of the 5 scenarios are fully satisfied for the second chance constraint set in Eq. 6 (CCP2), its actual probability (i.e., 100%) is larger than its target probability, i.e., 90%.

## 5.3 Computational analysis of the fuzzystochastic optimization results

In the fuzzy-stochastic optimization cases-1 and 2 ( $\alpha = 0$  and  $\alpha = 1$ ), all of the satisfied and unsatisfied scenarios with the machine availabilities are also presented in

Scenario No	Machin	ne Availal	pilities ( $\varphi_k$	)			CCP1	CCP2
	1	2	3	4	5 6			
Fuzzy-stochasti	c case-1 (	$\alpha = 0$ and	α = 1)					
1	А	А	А	А	А	А	Satisfied	Satisfied
2	NA	NA	А	А	А	А	Unsatisfied	Satisfied
3	А	А	А	А	А	А	Satisfied	Satisfied
4	А	А	А	А	А	А	Satisfied	Satisfied
5	А	А	NA	А	А	А	Satisfied	Satisfied
Fuzzy-stochasti	c case-2 (	$\alpha = 0$ and	$\alpha = 1$ )					
1	NA	А	А	NA	А	NA	Unsatisfied	Satisfied
2	А	А	А	А	А	А	Satisfied	Satisfied
3	А	А	NA	А	А	А	Satisfied	Satisfied
4	А	А	А	А	NA	А	Satisfied	Satisfied
5	А	А	А	А	А	А	Satisfied	Satisfied

 
 Table 8 Machine availabilities under different scenarios and the constraint satisfaction for medium overlap case

A Aavailable, NA not available

Table 9	<b>9</b> A detailed calculation	of the part	low-based layou	t objective in a deter	rministic optimiza	ution case	
REs	Occupied locations	RE2	RE3	RE4	RE5	RE6	Score of part flow rates
RE1	Location-1	0	0	$1 \times 0.02125$	$1 \times 0.02125$	$1 \times 0.13375$	0.51175

TCL5	occupied locations	ILE2	TCL0	RE I	RE5	ILL0	beore or p	are now rates
RE1	Location-1	0	0	1 × 0.02125	1 × 0.02125	1 × 0.13375	0.51175	
	Location-3	$1 \times 0.0145$	0	0	$1 \times 0.02125$	$1 \times 0.13375$	5	
	Location-5	0	$1 \times 0.011$	$1 \times 0.02125$	0	$1 \times 0.13375$	5	
		RE1	RE3	RE4	RI	E5	RE6	
RE2	Location-1	0	0	$1 \times 0.0$	32275 1	× 0.0145	$1 \times 0.055$	0.285075
	Location-2	$1 \times 0.02125$	$1 \times 0.0212$	5 $1 \times 0.0$	32275 0		0	
	Location-5	0	$1 \times 0.0212$	5 $1 \times 0.0$	32275 0		$1 \times 0.055$	
		RE1	RE2	RE4	RE	25	RE6	
RE3	Location-1	0	0	$1 \times 0$	0.011 1	× 0.075	$1 \times 0.0145$	0.3865
	Location-3	0	$1 \times 0.05$	5 0	1	× 0.075	$1 \times 0.0145$	
	Location-4	$1 \times 0.0115$	$1 \times 0.05$	5 0	1 :	× 0.075	0	
		RE1	RE2	RE3	RE	25	RE6	
RE4	Location-3	0	$1 \times 0.0212$	5 0	1 :	× 0.0323	$1 \times 0.011$	0.26385
	Location-4	$1 \times 0.055$	$1 \times 0.0212$	5 0	1 :	× 0.0323	0	
	Location-6	$1 \times 0.055$	$1 \times 0.0212$	5 $1 \times 0$ .	0145 0		0	
		RE1	RE2	RE3	RE	24	RE6	
RE5	Location-2	$1 \times 0.012$	0	$1 \times 0.$	0323 1 :	× 0.0145	0	0.24615
	Location-5	0	0	$1 \times 0.$	0323 1 :	× 0.0145	$1 \times 0.075$	
	Location-6	$1 \times 0.012$	$1 \times 0.0212$	5 $1 \times 0$ .	0323 0		0	
		RE1	RE2	RE3	RE4	Ļ	RE5	
RE6	Location-2	1 × 0.011	0	$1 \times 0.0$	955 1 ×	0.055	0	0.36
	Location-4	$1 \times 0.011$	$1 \times 0.075$	0	0		$1 \times 0.012$	
	Location-6	$1 \times 0.011$	$1 \times 0.075$	$1 \times 0.0$	055 0		0	
				Total pa	art flow-based lag	yout objective		2.0533
				Total di	Total distance-based layout objective			18
				Total la	yout score			20.0533

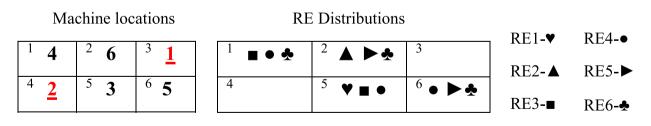


Fig. 4 Machine location assignment and RE distributions for fuzzy-stochastic case-1 in scenario-2 ( $\alpha = 0$ )

Table 8 for the medium overlap case only. According to this table, CCP1 is unsatisfied in scenario-2 of fuzzystochastic optimization case-1, where machines-1 and 2 are broken down at the same time. Since the unsatisfied scenario-2 is compatible with the unavailability of machines-1

and 2, these machines can be described as critical machines, which necessitate preventive maintenance actions to avoid their simultaneous downtime. On the other hand, both CCP1 and CCP2 are satisfied in scenario-5 despite the unavailability of machine-3. Thus, the generated robust

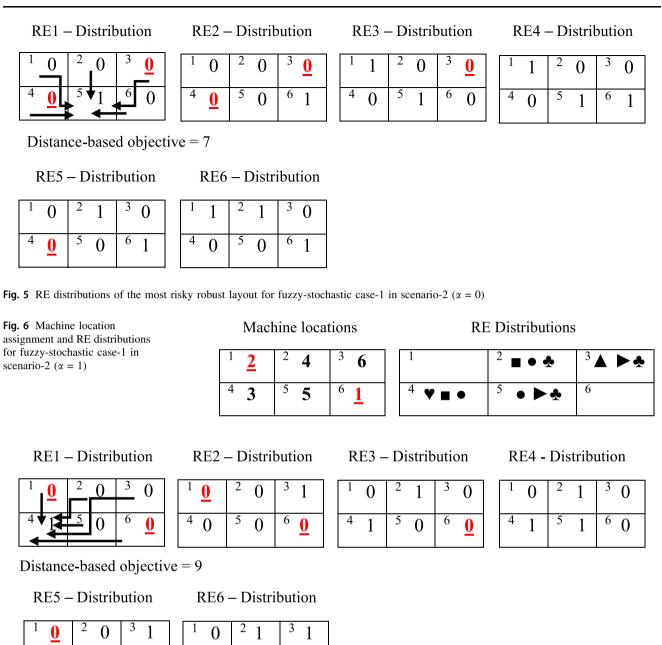


Fig. 7 RE distributions of the risk-free robust layout for fuzzy-stochastic case-1 in scenario-2 ( $\alpha = 1$ )

5 1

4

0

Fig. 8 Machine location assignment and RE distributions for medium overlap case in deterministic optimization

5

1

<sup>6</sup> 0

4

0

## Machine locations

<sup>6</sup> 0

**RE** Distributions

		3♥∎●
4 ∎ ● ♣	<sup>5</sup> <b>V A &gt;</b>	<sup>6</sup> ● ▶ ♣

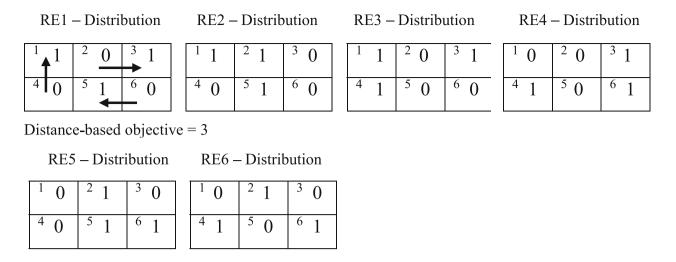


Fig. 9 RE distributions of the generated layout for the medium overlap case in deterministic optimization

layout is able to satisfy all of the probabilistic constraints despite the absence of this non-critical machine-3. In a similar manner, the instantaneous unavailability of the critical machines-1, 4, and 6 caused the unsatisfied scenario-1 in the fuzzy-stochastic optimization case-2. On the other hand, the unavailability of non-critical machines-3 and 5 doesn't cause any constraint violation since their alternative machines have the same processing capabilities and are not broken down.

As mentioned before, when a machine is broken down, all of its processing capabilities (or REs) are disappeared from its location. For instance, the machine location assignments of the most risky robust layout design under scenario-2 ( $\alpha = 0$ ) and its relevant RE distributions are displayed in Figs. 4 and 5, respectively. Similarly, the risk-free robust layout design with its RE distributions is also depicted in Figs. 6 and 7 for the same fuzzy-stochastic optimization case-1 ( $\alpha = 1$ ). It should be noted here that the machines-1 and 2 are broken down in both of these risky and risk-free robust layout designs. Moreover, as clearly seen in Figs. 4 and 6, the produced robust layout designs are completely different from each other under different uncertainty levels of the fuzzy part flow rates.

Therefore, although the fuzzy part flow rates have not a significant effect on the layout score, different uncertainty levels of fuzzy part flows may change the robust layout design completely. This is the reason why we took into account the fuzzy part flow rates with such an  $\alpha$ -parametric fuzzy resolution approach. To compare the results of

deterministic and fuzzy-stochastic optimization cases, the proposed approach is also applied without considering any type of uncertainty. The resulting machine locations assignments and the relevant RE distributions are depicted in Figs. 8, 9 for the medium capability overlap case in deterministic optimization.

In this deterministic optimization case, all of the machines are assumed to be available (or not broken down) and the part flows rates are known exactly. In this situation, the distance-based objective is equal to 3 for each RE distribution as shown in Fig. 9 since there is only one unit distance from each unoccupied location to its nearest occupied location by the relevant REs. Therefore, the total distance-based layout objective is equal to 18 ( $6 \times 3$ ) when all the REs are considered. Similarly, the calculation of this distance-based layout score has been already illustrated based on RE-1 in Figs. 5 and 7 for the fuzzy-stochastic optimization cases.

Additionally, a detailed calculation of the part flowbased layout objective is shown in Table 9 for the deterministic optimization case. It should be noted here that while calculating this part flow-based layout objective, the expected values of triangular fuzzy numbers are used to obtain crisp values for the part flow rates (see Eq. 19). It is seen in Table 9 that the total layout score, which is equal to the sum of the total distance and part flow-based layout objectives, is equal to 20.0533 which has already been found by LINGO 19.0 solver for the medium capability overlap case (see Table 7). The calculation of the part flow**Table 10** Calculation of the part flow-based layout objective for fuzzy-stochastic case-1 in scenario-2 ( $\alpha = 0$ )

REs	Occupied locat	ions RE2	RE3	RE4	RE5	RE6		Score of part	flow rates
RE1	Location-5	$1 \times 0.0$	013 0	0	$1 \times 0.02$	2 1 ×	0.1295	0.1625	
		RE1		RE3		RE4	RE5	RE6	
RE2	Location-6	6 1 × 1	0.02	$1 \times 0.$	02	0	0	0	0.04
		RE1	RE2		RE4	RE5		RE6	
RE3	Location-1	2 × 0.01	$3 \times 0$	.0515	0	1 ×	0.0725	0	0.384
	Location-5	0	$1 \times 0$	.0515	0	$1 \times$	0.0725	$1 \times 0.013$	
		RE1	RE2	R	E3	RE5		RE6	
RE4	Location-1 Location-5	$\begin{array}{c} 2 \times 0.0515 \\ 0 \end{array}$	$3 \times 0.0$ $1 \times 0.0$				0.0321 0.0321	0 1 × 0.0105	0.3222
	Location-6	$1 \times 0.0515$	0		× 0.013	0	0.0321	0	
		RE1	RE2		RE3		RE4	RE6	
RE5	Location-2 Location-6	$1 \times 0.010$ $1 \times 0.010$		0.02	$1 \times 0.0$ $1 \times 0.0$		$1 \times 0.0$	013 0 0	0.1382
		RE1	RE2	RI	E3	RE4		RE5	
RE6		$2 \times 0.0105$ $1 \times 0.0105$	$3 \times 0.072$ $2 \times 0.072$		× 0.0515	0	0.0515	$1 \times 0.0105$ 0	0.5695
		$1 \times 0.0105$ $1 \times 0.0105$	2 × 0.072		$\times 0.0515$ $\times 0.0515$	$1 \times 0$	0.0313	0	
			-	Тс	otal distan	ce-based	•	objective objective	1.6164 32
				To	otal layou	t score			33.6164

based layout objective is also presented in Table 10 for the fuzzy-stochastic optimization case-1 under scenario-2 ( $\alpha = 0$ ).

As an expected result, the part flow-based layout objective is comparatively high (i.e., 33.6164) due to the simultaneous unavailability of machines-1 and 2 (see Table 10). On the other hand, the part flow-based layout score (i.e., 19.9072) will decrease considerably under scenarios-1, 3, and 4 ( $\alpha = 0$ ) (see Table 11) because none of the machines are broken down in these scenarios. The final layout score that is already given in Table 7 (i.e., 24.0983) will be the expected values of these 5 scenarios with equal probabilities (i.e., 20%). As it is also obviously seen in Tables 7, 10, 11, the total layout score took values between the minimum (i.e., 19.9072) and maximum (i.e., 33.6164) layout scores under different scenarios.

Finally, in order to analyze the effects of different machine availabilities (%) and the satisfaction probabilities

for the chance constraint sets, additional computational analyses are carried out and presented in Tables 12, 13. In Table 12, the machine availability rates are reduced to 85% and the target probability of CCP1 is increased up to 90%. In this circumstance, any feasible solution couldn't be found for the low and medium capability overlap cases similar to the no overlap case. However, we could achieve the optimal solutions for these cases by using the formerly defined probability values given in Table 7. According to Tables 7 and 12, infeasible or feasible solutions but comparatively higher layout scores will be provided if one decreases the machine availabilities and increases the satisfaction probabilities of the chance constraint sets. It should also be noted here that the number of infeasible solutions will increase and the layout scores will deteriorate in case of the binomially distributed random machine breakdowns. Fortunately, fewer amount of infeasible solutions and comparatively better layout scores can be

Table 11	Calculation of part flow	v-based objective	for fuzzy-stochas	stic case-1 in sc	enarios-1, 3, ar	and 4 ( $\alpha = 0$ )	
REs	Occupied locations	RE2	RE3	RE4	RE5	RE6	Score of part flow rates
RE1	Location-3	0	0	$1 \times 0.02$	$1 \times 0.02$	$1 \times 0.1295$	0.492

RE1	Location-3	0	0	$1 \times 0.02$ 1	$\times 0.02$ 1 $\times 0.129$	95 0.492	
	Location-4	0	$1 \times 0.0105$	$1 \times 0.02$ 0	$1 \times 0.129$	95	
	Location-5	$1 \times 0.013$	0	0 1	$\times 0.02$ 1 $\times 0.129$	95	
		RE1	RE3	RE4	RE5	RE6	
RE2	Location-3	0	0	1 × 0.0321	$1 \times 0.013$	$1 \times 0.0515$	0.2402
	Location-4	0	$1 \times 0.02$	$1 \times 0.0321$	0	$1 \times 0.0515$	
	Location-6	$1 \times 0.02$	$1 \times 0.02$	0	0	0	
		RE1	RE2	RE4	RE5	RE6	
RE3	Location-1	$1 \times 0.01$	$1 \times 0.0515$	0	$1 \times 0.0725$	0	0.367
	Location-3	0	0	$1 \times 0.0105$	$1 \times 0.0725$	$1 \times 0.013$	
	Location-5	0	$1 \times 0.0515$	0	$1 \times 0.0725$	$1 \times 0.013$	
		RE1	RE2	RE3	RE5	RE6	
RE4	Location-1	$1 \times 0.0515$	$1 \times 0.02$	0	$1 \times 0.0321$	0	0.2307
	Location-5	0	$1 \times 0.02$	0	$1 \times 0.0321$	$1 \times 0.0105$	
	Location-6	$1 \times 0.0515$	0	1 × 0.013	0	0	
		RE1	RE2	RE3	RE4	RE6	
RE5	Location-2	$1 \times 0.0105$	$1 \times 0.02$	$1 \times 0.0321$	$1 \times 0.013$	0	0.2358
	Location-4	0	0	$1 \times 0.0321$	$1 \times 0.013$	$1 \times 0.0725$	
	Location-6	$1 \times 0.0105$	0	$1 \times 0.0321$	0	0	
		RE1	RE2	RE3	RE4	RE5	
RE6	Location-1	$1 \times 0.0105$	$1 \times 0.0725$	0	0	$1 \times 0.0105$	0.3415
	Location-2	$1 \times 0.0105$	$1 \times 0.0725$	$1 \times 0.0515$	$1 \times 0.0515$	0	
	Location-6	$1 \times 0.0105$	0	$1 \times 0.0515$	0	0	
				Total part flow	v-based layout objectiv	e	1.9072
				Total distance-	Total distance-based layout objective		18
				Total layout so	core		19.9072

obtained when the random machine breakdowns are defined as basic discrete variables with known probabilities. Similarly, computational analyses are also carried out by reducing the target probabilities of the chance constraint sets (CCP1 and CCP2) to 60% under the same machine availabilities (i.e., 85%). In this case, feasible and comparatively better layout scores are provided as in Table 13 for all of the capability overlap cases. However, it should be emphasized here that higher satisfaction probabilities

are generally desired by the facility designers for these chance constraint sets. Lastly, feasible solutions couldn't be found for the no overlap case in fuzzy-stochastic optimization case-2. Moreover, it is clearly seen in Table 13 that the usage of discrete random variables with known probabilities for the uncertain machine unavailability may provide better quality layout scores when compared to the binomial distribution.

 Table 12 Computational analysis on the fuzzy-stochastic optimization cases

		No overlap	Low overlap	Medium overlap	High overlap	Very high overlap	Fully overlap
Fuzzy-stochastic case-1 ( $\alpha = 0$ )	Overall layout score	Infeasible	Infeasible	Infeasible (N.A)	39.5289	33.8585	27.8352
	# of unsatisfied scenarios for CCP1	5	5	5	0	0	0
	Actual probability for CCP1	0%	0%	0%	100%	100%	100%
	# of unsatisfied scenarios for CCP2	5	5	5	0	0	0
	Actual probability for CCP2	0%	0%	0%	100%	100%	100%
	CPU time (sec.)	1.17	1.52	9.0	10.23	14.36	3.23
	Total solver iterations	0	0	72,876	166,300	162,638	50,942
	Extended solver steps	0	0	2	537	1001	1400
Fuzzy-stochastic case-2	Overall layout score	Infeasible	Infeasible	Infeasible	Infeasible	Infeasible	48.7116
$(\alpha = 0)$	# of unsatisfied scenarios for CCP1	5	5	5	5	5	0
	Actual probability for CCP1	0%	0%	0%	0%	0%	100%
	# of unsatisfied scenarios for CCP2	5	5	5	5	5	0
	Actual probability for CCP2	0%	0%	0%	0%	0%	100%
	CPU time (sec.)	1.37	1.32	5.45	5.5	14.51	2.42
	Total solver iterations	0	0	50,276	61,089	174,978	24,083
	Extended solver steps	0	0	2	0	631	352
Fuzzy-stochastic case-1	Overall layout score	Infeasible	Infeasible	Infeasible	39.91125	34.2868	28.2458
$(\alpha = 1)$	# of unsatisfied scenarios for CCP1	5	5	5	0	0	0
	Actual probability for CCP1	0%	0%	0%	100%	100%	100%
	# of unsatisfied scenarios for CCP2	5	5	5	0	0	0
	Actual probability for CCP2	0%	0%	0%	100%	100%	100%
	CPU time (sec.)	1.86	1.44	6.21	10.34	10.14	1.9
	Total solver iterations	0	0	69,228	143,111	164,432	25,706
	Extended solver steps	0	0	2	932	734	446
Fuzzy-stochastic case-2	Overall layout score	Infeasible	Infeasible	Infeasible	Infeasible	Infeasible	49.43015
$(\alpha = 1)$	# of unsatisfied scenarios for CCP1	5	5	5	5	5	0
	Actual probability for CCP1	0%	0%	0%	0%	0%	100%
	# of unsatisfied scenarios for CCP2	5	5	5	5	5	0
	Actual probability for CCP2	0%	0%	0%	0%	0%	100%
	CPU time (sec.)	1.54	1.33	2.62	5.27	12.75	2.04
	Total solver iterations	0	0	7211	76,568	178,399	34,337
	Extended solver steps	0	0	0	0	507	197

Sample size = 5; machine availabilities = 85%; probability for breakdowns = 15%; probabilities for the chance-constraint sets = 90%

Table 13 Computational analysis on the fuzzy-stochastic optimization cases

		No overlap	Low overlap	Medium overlap	High overlap	Very high overlap	Fully overlap
Fuzzy-stochastic case-1	Overall layout score	51.3365	25.6386	19.9593	13.6331	6.9588	0
$(\alpha = 0)$	# of unsatisfied scenarios for CCP1	2	2	2	2	2	2
	Actual probability for CCP1	60%	60%	60%	60%	60%	60%
	# of unsatisfied scenarios for CCP2	1	1	1	1	1	1
	Actual probability for CCP2	80%	80%	80%	80%	80%	80%
	CPU time (sec.)	8.83	1.26	9.48	6.89	7.39	0.26
	Total solver iterations	93,373	12,557	149,735	55,910	42,653	0
	Extended solver steps	533	222	3723	3737	4646	0
Fuzzy-stochastic case-2 (α = 0)	Overall layout score	Infeasible	31.8928	24.0983	18.0151	12.6214	6.9588
	# of unsatisfied scenarios for CCP1	5	2	2	2	2	2
	Actual probability for CCP1	0%	60%	60%	60%	60%	60%
	# of unsatisfied scenarios for CCP2	5	0	0	0	0	0
	Actual probability for CCP2	0%	100%	100%	100%	100%	100%
	CPU time (sec.)	1.92	16.55	16.64	28.23	44.66	1.68
	Total solver iterations	0	171,532	200,260	289,808	391,867	10,462
	Extended solver steps	0	417	1441	10,255	36,953	220
Fuzzy-stochastic case-1	Overall layout score	51.49245	25.8089	20.14735	13.7959	7.06145	0
$(\alpha = 1)$	# of unsatisfied scenarios for CCP1	2	2	2	2	2	2
	Actual probability for CCP1	60%	60%	60%	60%	60%	60%
	# of unsatisfied scenarios for CCP2	1	1	1	1	1	1
	Actual probability for CCP2	80%	80%	80%	80%	80%	80%
	CPU time (sec.)	10.26	1.09	6.07	23.19	10.9	0.26
	Total solver iterations	111,489	11,389	114,267	305,812	125,530	0
	Extended solver steps	867	163	4202	22,346	11,579	0
Fuzzy-stochastic case-2	Overall layout score	Infeasible	32.09085	24.30785	18.2229	12.7989	7.06145
$(\alpha = 1)$	# of unsatisfied scenarios for CCP1	5	2	2	2	2	2
	Actual probability for CCP1	0%	60%	60%	60%	60%	60%
	# of unsatisfied scenarios for CCP2	5	0	0	0	0	0
	Actual probability for CCP2	0%	100%	100%	100%	100%	100%
	CPU time (sec.)	2.91	5.38	34.23	25.49	52.96	1.53
	Total solver iterations	0	62,048	364,527	282,836	466,142	9615
	Extended solver steps	0	140	3121	9457	42,259	240

Sample size = 5; machine availabilities = 85%; probability for breakdowns = 15%; probabilities for the chance-constraint sets = 60%

## 6 A real-life application study

In order to display the usefulness, validity, and practicality of the proposed robust biased CBDL approach, an application study is presented for a real-life cellular manufacturing system which was previously introduced by Baykasoğlu and Gindy (2000) to the literature.

#### 6.1 Data description and company information

The manufacturing company receives various customer orders with relatively low demand quantities and commonly encounters with layout design changes. Since the highly uncertain nature of the customer demand, they cannot estimate the demand quantities accurately. For that

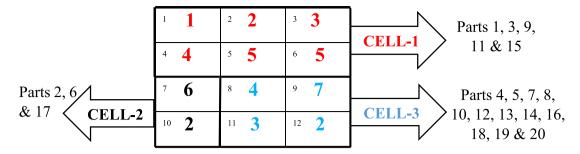


Fig. 10 Existing cellular layout of the manufacturing facility with part assignments

Table 14 Fuzzy demand quantities and the process flow information

Parts	(Demand $\times$ 1000)	Processing routes
1	(1, 3, 5)	RE1-RE2-RE4
2	(0.3, 1, 2)	RE1-RE2-RE3
3	(0.5, 2.5, 4)	RE5-RE6-RE7
4	(0.52, 1.52, 2.52)	RE8-RE5
5	(0.48, 1.48, 2.18)	RE7-RE4-RE5
6	(1.2, 3.5, 5.5)	RE8-RE6-RE7
7	(0.5, 1, 1.5)	RE8-RE9-RE10
8	(0.5, 2, 3)	RE9-RE10-RE11
9	(1, 3, 5)	RE5-RE1-RE2
10	(0.8, 2, 3)	RE3-RE4
11	(1.8, 4.5, 7)	RE5-RE6-RE9
12	(0.3, 1, 2)	RE10-RE9-RE8
13	(1, 3, 5)	RE5-RE8-RE10
14	(1, 2.5, 4)	RE8-RE7-RE5
15	(1, 2.5, 3.5)	RE1–RE2
16	(0.5, 1.9, 2.7)	RE3-RE4
17	(1, 2.4, 3.4)	RE6-RE7-RE8
18	(0.6, 1.2, 2.5)	RE8-RE9-RE10-RE11
19	(0.3, 1.3, 2.3)	RE5-RE2
20	(1, 3, 5)	RE7-RE8-RE9

reason, facility managers determined to describe fuzzy demand quantities for these customer orders. In the present case, the facility floor is organized as a classical cellular manufacturing environment without considering the machines' processing capabilities. The  $4 \times 3$  block plan and the existing cellular layout of the facility have already been shown in Baykasoğlu and Gindy (2000) as depicted in Fig. 10. In detail, there are 3 manufacturing cells and 7 different machines inside the factory. However, some machines have multiple copies. Therefore, there are a total of 12 machines and 12 locations available on the factory floor where copied machines include the same REs. The manufactured parts in each cell are also shown in Fig. 10.

Because of the extensive material handling operations between different cells and low cell capacity utilizations,

the facility managers are not fully satisfied with the performance of the existing cellular layout because non-valueadded material handling operations between different cells are needed for some parts. This may also cause extra machine requirements. For instance, one more copy of machine-2 is required for cell-2 and two more copies of machine-2 are needed for cell-3. This leads to significant time and money consumption, which also decreases the efficiency and profitability of the manufacturing company. Furthermore, the capacity utilizations are calculated for each cell by including the extra copies of the required machines as 61%, 57%, and 72%, respectively. Therefore, the facility managers seek a better robust layout alternative, which improves the efficiency of the material handling operations and increases the machine capacity utilization by considering the machines' processing capabilities (or REs), unavailability risks, and uncertainty in the part flow rates. To do this, the proposed R-CBDL approach was considered as an alternative method to obtain optimal product flows between different departments under uncertainty. Before applying the proposed approach, 11 different REs are first specified by the facility process planners for the machines' processing capabilities. Thereafter, the uncertain demand quantities for the customer orders are represented by using triangular fuzzy numbers as given in Table 14. In this table, in addition to the monthly fuzzy demand quantities, the processing requirements (or process flow information) of a total of 20 manufactured parts are also identified in terms of the REs. Furthermore, the number of copies for available machines with their processing capability information (i.e., REs) is shown in Table 15. The probabilities for the machine availabilities that are calculated based on the previous machine breakdown statistics of the facility reports are also given in Table 15.

Through the fuzzy demand quantities and process flow information in Table 14, the fuzzy transition rate matrix between different REs is calculated by using Eq. (11) as in Table 16. It should be noted here that the sum of all the transition rates is also equal to a fuzzy number, i.e., (0.956, 1.001, 1.056) which already contains "1". Finally, the

Machines	Copies	Resource elements (REs)									Machine availability		
		1	2	3	4	5	6	7	8	9	10	11	(%)
Drill Press-1 (M1)	1	*											95
MHP Machining Centre-1 (M2-M3- M4)	3	*	*	*				*	*	*	*	*	90
Colchester Lathe-1 (M5-M6)	2	*	*		*			*					85
MHP MT50 NC Lathe-2 (M7-M8)	2	*	*		*			*			*		85
CNC Grinding Machine-1 (M9-M10)	2					*	*						90
Jones and Shipman Cyc. Grinder (M11)	1						*						95
Jones * Ship. Surf. Grinder (M12)	1					*							95

Table 16 Fuzzy transition rate matrix between different machining capabilities (or REs) for the real-life case study

	RE1	RE2	RE3	RE4	RE5	RE6	RE7	RE8	RE9	RE10	RE11
RE1	_	(0.117, 0.118, 0.119)									
RE2		-	(0.011, 0.012, 0.015)	(0.036, 0.037, 0.038)							
RE3			-	(0.044, 0.046, 0.048)							
RE4				-	(0.017, 0.018, 0.019)						
RE5	(0.036, 0.037, 0.038)	(0.011, 0.016, 0.018)			_	(0.082, 0.084, 0.087)		(0.036, 0.037, 0.038)			
RE6						-	(0.096, 0.099, 0.104)		(0.054, 0.056, 0.064)		
RE7				(0.016, 0.017, 0.018)	(0.031, 0.036, 0.037)		-	(0.064, 0.067, 0.071)			
RE8					(0.017, 0.018, 0.019)	(0.042, 0.043, 0.044)	(0.032, 0.036, 0.037)	-	(0.065, 0.069, 0.075)	(0.036, 0.037, 0.038)	
RE9								(0.011, 0.012, 0.015)	-	(0.052, 0.054, 0.057)	
RE10									(0.011, 0.012, 0.015)	-	(0.039, 0.040, 0.042)
RE11											_

sample size (or the number of scenarios) is set to 5 and the target probabilities of the chance constraint sets are specified as 80% by the facility managers. The facility

managers preferred to use discrete random variables with known probabilities since the binomial distribution may

 Table 17
 Fuzzy-stochastic

 optimization results under
 different uncertainty levels

$\alpha$ -feasibility degree of CCP2	$\alpha = 0$	$\alpha = 0.3$	$\alpha = 0.5$	$\alpha = 0.7$	$\alpha = 1$
Total random variables	12				
Total integer variables (Deteq.)	1738				
Total constraints (Deteq.)	4383				
Total nonzeros (Deteq.)	39,850				
Total solver iterations	89,593,646	103,498,008	146,909,359	89,344,547	97,063,615
Extended solver steps	720,796	694,363	968,737	416,112	413,412
CPU time (sec.)	10,800	10,800	10,800	10,800	10,800
Layout score (Obj. value)	144.866	145.177	145.3622	145.5363	145.7975
Optimality gap	12%	18%	21%	19%	16%
# of unsatisfied scenarios (CCP1)	1	1	1	1	1
Actual probability (CCP1)	80%	80%	80%	80%	80%
Unsatisfied scenarios (CCP2)	0	0	0	0	0
Actual probability (CCP2)	100%	100%	100%	100%	100%

Table 18	Machine availabilities
under diffe	erent scenarios and
the resulting	ng constraint
satisfaction	$\alpha = 0 \text{ and } 1$

Scenario No	Ma	Machine Availabilities $(\varphi_k)$											CCP1	CCP2
	1	2	3	4	5	6	7	8	9	10	11	12		
1	А	А	А	А	А	А	NA	NA	А	А	А	А	Satisfied	Satisfied
2	А	NA	NA	А	А	А	А	А	А	А	А	А	Unsatisfied	Satisfied
3	А	А	А	А	А	NA	А	А	А	А	А	А	Satisfied	Satisfied
4	А	А	А	А	А	А	А	А	А	А	А	А	Satisfied	Satisfied
5	А	А	А	NA	А	А	А	А	А	А	А	А	Satisfied	Satisfied

result in worse layout scores as discussed previously in Sect. 5.

# 6.2 Details of the fuzzy-stochastic optimization results

By using the given data for this case study, the proposed fuzzy-stochastic programming model is run under different uncertainty levels ( $\alpha$ -cuts) and probabilistic scenarios with the help of LINGO 19.0 optimization software on an Intel Corei7 2 GHz IBM PC. The details of the fuzzy-stochastic optimization results within the 3-h runtime limit are provided as in Table 17. According to this table, the first chance constraint set in Eq. 5 is satisfied with 80% actual probability, whereas the second chance constraint set in Eq. 6 is satisfied with 100% actual probability under all uncertainty levels. This means that only one scenario may cause unsatisfactory solutions because of the random machine breakdowns. It is also clearly seen in Table 17 that the layout score increases when the feasibility degree of the fuzzy-stochastic constraint set (CCP2) gets higher. Actually, this is an expected result since the risk-free layout design options may generally result in more costly solutions. It should be highlighted here again that the optimal layout scores are not acquired within the 3-h runtime limit. However, the generated layout design alternatives under all uncertainty levels can be stated as applicable since their optimality gaps are low enough (or within acceptable levels). The details of the satisfied/unsatisfied scenarios with the machine statue (available or broken down) are also presented in Table 18. It should be noted here that the same machine availabilities and the resulting constraint satisfaction are yielded under different scenarios for both the most risky ( $\alpha = 0$ ) and risk-free ( $\alpha = 1$ ) cases.

It is also seen in Table 18 that MHP Machining Center-1 has critical machines that necessitate effective preventive maintenance actions. In other words, the simultaneous breakdown of the machines-2 and 3 (M2 and M3 in MHP Machining Center-1) may cause unsatisfied CCP1 (scenario#2), which violates the first probabilistic constraint set (see Tables 17, 18). Although this machining center has three copies of the same machines, the simultaneous breakdown of two of them may violate the first chance constraint set. It should also be noted here that if all the machines (M2, M3 and M4) in this MHP Machining Center-1 are broken down, the proposed fuzzy-stochastic programming model cannot provide any feasible solution because some of the REs, i.e., RE3, RE8, RE9, and RE11, are covered by only MHP Machining Center-1 (see Table 15). Thus, the simultaneous breakdown of three of them will cause an infeasible solution since it is not possible to reach these machining capabilities or REs from any

#### Machine locations

1 6	<sup>2</sup> 2	<sup>3</sup> 2
4 7	5 3	<sup>6</sup> 5
7 4	<sup>8</sup> 2	<sup>9</sup> 4
10 3	11 5	12 1

RE	E distribution	IS
1	<sup>2</sup> 🌲 📥	<sup>3</sup> 📥 🔺
	♥☆♦●♦	¥☆♦●
4 Δ	5 👲 🔺	<sup>6</sup> Δ□
	▶ ♥	
7 📥 🔺	<sup>8</sup> ♣ ▲ ■	9 📥 🔺
▶♥•	♥☆♦●◊	
10 🔶 🔺	$^{11}$ $\Delta \square$	12 📥
▶ ♥		

	RE1 秦	RE7 ♥
$\diamond$	RE2	RE8 🔅
	RE3	RE9 ♦
	RE4 🕨	RE10 •
	RE5 $\Delta$	RE11 🛇
	RE6 □	

Fig. 11 Machine location assignment and RE distributions for the most risky case under scenario#4 ( $\alpha = 0$ )

RE1 – Distribution

<sup>1</sup> 0	<sup>2</sup> 1	<sup>3</sup> 1
4 0	<sup>5</sup> 1	<sup>6</sup> 0
7 1	<sup>8</sup> 1	<sup>9</sup> 1
<sup>10</sup> 1	<sup>11</sup> 0	<sup>12</sup> 1

RE5 – Distribution

1 0	<sup>2</sup> 0	<sup>3</sup> 0
4 1	<sup>5</sup> 0	<sup>6</sup> 1
7 0	<sup>8</sup> 0	<sup>9</sup> 0
<sup>10</sup> 0	<sup>11</sup> 1	<sup>12</sup> 0

RE9 - Distribution

<sup>1</sup> 0	<sup>2</sup> 1	<sup>3</sup> 1
4 0	<sup>5</sup> 0	<sup>6</sup> 0
7 0	<sup>8</sup> 1	<sup>9</sup> 0
<sup>10</sup> 0	<sup>11</sup> 0	<sup>12</sup> 0

RE2 – Distribution	1

<sup>1</sup> 0	<sup>2</sup> 1	<sup>3</sup> 1
4 0	<sup>5</sup> 1	<sup>6</sup> 0
7 1	<sup>8</sup> 1	<sup>9</sup> 1
<sup>10</sup> 1	<sup>11</sup> 0	<sup>12</sup> 0

RE6 – Distribution

 $^{2}$  0 3 0 1 1 4 0 <sup>5</sup> 0 6 1 8 7 0 0 <sup>9</sup> 0 10 012 0<sup>11</sup> 1

ion

1 0 9 8 1 1 12010 0<sup>11</sup> 0

**RE3** – Distribution

1 0	<sup>2</sup> 1	<sup>3</sup> 1
4 0	<sup>5</sup> 0	<sup>6</sup> 0
7 0	<sup>8</sup> 1	<sup>9</sup> 0
<sup>10</sup> 0	<sup>11</sup> 0	<sup>12</sup> 0

RE7 – Distribution

<sup>1</sup> 0	<sup>2</sup> 1	<sup>3</sup> 1
4 0	<sup>5</sup> 1	<sup>6</sup> 0
7 1	<sup>8</sup> 1	<sup>9</sup> 1
<sup>10</sup> 1	<sup>11</sup> 0	<sup>12</sup> 0

RE4 – Distribution
--------------------

1 0	<sup>2</sup> 0	<sup>3</sup> 0
4 0	5 1	<sup>6</sup> 0
7 1	8 0	<sup>9</sup> 1
10 1	<sup>11</sup> 0	<sup>12</sup> 0

RE8 – Distribution

1 0	<sup>2</sup> 1	<sup>3</sup> 1
4 0	<sup>5</sup> 0	<sup>6</sup> 0
7 0	<sup>8</sup> 1	<sup>9</sup> 0
10 0	<sup>11</sup> 0	<sup>12</sup> 0

	R	E10	_ ]	Distril	outi
	1	0	2	1	3
	4	0	5	0	6

RE11 – Distribution

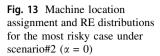
1 0	<sup>2</sup> 1	<sup>3</sup> 1
4 0	<sup>5</sup> 0	<sup>6</sup> 0
7 0	<sup>8</sup> 1	<sup>9</sup> 0
<sup>10</sup> 0	<sup>11</sup> 0	<sup>12</sup> 0

Fig. 12 RE distributions of the produced layout design for the most risky case under scenario#4 ( $\alpha = 0$ )

factory location. On the other hand, the simultaneous breakdown of non-critical machines M7 and M8 (MHP MT50 NC Lathe-2) doesn't lead to any unsatisfied scenario (see scenario#1) since the REs (i.e., RE1, RE2, RE4, RE7, and RE10) that are covered by these machines have already

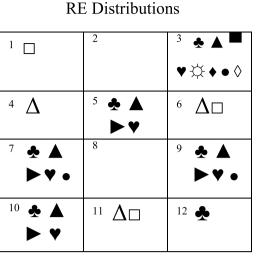
been included by other alternative machines like MHP MT50 NC Lathe-2 and MHP Machining Center-1.

Finally, the produced most risky robust layout design alternative ( $\alpha = 0$ ) including the machine location assignment and the relevant RE distributions is demonstrated in Figs. 11 and 12 under scenario#4 where all of the machines



Mac	hine	locati	ons

<sup>1</sup> 6	<sup>2</sup> <b>2</b>	<sup>3</sup> 2
4 7	5 3	<sup>6</sup> 5
74	<sup>8</sup> 2	<sup>9</sup> 4
10 3	11 5	<sup>12</sup> 1



1 0	<sup>2</sup> <b>0</b>	<sup>3</sup> 1
4 0	<sup>5</sup> 1	<sup>6</sup> 0
7 1	<sup>8</sup> <u>0</u>	<sup>9</sup> 1
10 1	<sup>11</sup> 0	<sup>12</sup> 1

RE5-Distribution

1 0	<sup>2</sup> <u>0</u>	<sup>3</sup> 0
4 1	<sup>5</sup> 0	<sup>6</sup> 1
7 0	<sup>8</sup> <u>0</u>	<sup>9</sup> 0
<sup>10</sup> 0	<sup>11</sup> 1	<sup>12</sup> 0

RE9 – Distribution

<sup>1</sup> 0	<sup>2</sup> <u>0</u>	<sup>3</sup> 1
4 0	<sup>5</sup> 0	<sup>6</sup> 0
7 0	<sup>8</sup> <u>0</u>	<sup>9</sup> 0
<sup>10</sup> 0	<sup>11</sup> 0	<sup>12</sup> 0

<sup>1</sup> 0	<sup>2</sup> <b>0</b>	<sup>3</sup> 1
4 0	<sup>5</sup> 1	<sup>6</sup> 0
7 1	<sup>8</sup> <u>0</u>	<sup>9</sup> 1
10 1	<sup>11</sup> 0	<sup>12</sup> 0

RE2 – Distribution

RE6 – Distribution

<sup>1</sup> 1	<sup>2</sup> <u>0</u>	<sup>3</sup> 0
4 0	<sup>5</sup> 0	<sup>6</sup> 1
7 0	<sup>8</sup> <u>0</u>	<sup>9</sup> 0
<sup>10</sup> 0	<sup>11</sup> 1	<sup>12</sup> 0

RE10 – Distribution

1 0	<sup>2</sup> <u>0</u>	<sup>3</sup> 1
4 0	<sup>5</sup> 0	<sup>6</sup> 0
7 1	<sup>8</sup> <u>0</u>	<sup>9</sup> 1
<sup>10</sup> 0	<sup>11</sup> 0	<sup>12</sup> 0

RE3 –	Distribution
NEJ –	Distribution

<sup>1</sup> 0	<sup>2</sup> <b>0</b>	<sup>3</sup> 1
4 0	<sup>5</sup> 0	<sup>6</sup> 0
7 0	<sup>8</sup> <u>0</u>	<sup>9</sup> 0
<sup>10</sup> 0	<sup>11</sup> 0	<sup>12</sup> 0

RE7 – Distribution

1 0	<sup>2</sup> <b><u>0</u></b>	<sup>3</sup> 1
4 0	<sup>5</sup> 1	<sup>6</sup> 0
7 1	<sup>8</sup> <u>0</u>	<sup>9</sup> 1
<sup>10</sup> 1	<sup>11</sup> 0	<sup>12</sup> 0

RE4 – Distribution	
--------------------	--

1 0	<sup>2</sup> <u>0</u>	<sup>3</sup> 0
4 0	<sup>5</sup> 1	<sup>6</sup> 0
7 1	<sup>8</sup> <u>0</u>	<sup>9</sup> 1
10 1	<sup>11</sup> 0	<sup>12</sup> 0

RE8 – Distribution

1 0	<sup>2</sup> <u>0</u>	<sup>3</sup> 1
4 0	<sup>5</sup> 0	<sup>6</sup> 0
7 0	<sup>8</sup> <u>0</u>	<sup>9</sup> 0
<sup>10</sup> 0	$^{11}$ 0	<sup>12</sup> 0

RE11 – Distribution	RE1	1 —	Dis	trib	ution	
---------------------	-----	-----	-----	------	-------	--

1 0	<sup>2</sup> <b>0</b>	<sup>3</sup> 1
4 0	<sup>5</sup> 0	<sup>6</sup> 0
7 0	<sup>8</sup> 0	<sup>9</sup> 0
<sup>10</sup> 0	<sup>11</sup> 0	<sup>12</sup> 0

Fig. 14 RE distributions of the produced layout design for the most risky case under scenario#2 ( $\alpha = 0$ )

are available (or not broken down). It is also clearly seen in Table 18 that all of the probabilistic constraint sets are satisfied under scenario#4. As displayed in Figs. 11, 12,

since none of the machines are broken down under scenario#4, all of their processing capabilities appeared in the relevant RE distribution table (or binary matrix). In the RE

#### Fig. 15 Machine location assignment and RE distributions for the risk-free case under scenario#4 ( $\alpha = 1$ )

Machina	locations
Machine	locations

	-	-
1 1	<sup>2</sup> 5	<sup>3</sup> 3
4 4	5 2	<sup>6</sup> 4
7 6	8 3	<sup>9</sup> 5
10 7	11 2	<sup>12</sup> 2

1 📥	<sup>2</sup> $\Delta$ $\Box$	<sup>3</sup> 🛧 🔺
		▶ ♥
4 🛧 🛦	5 🔶 🔺	<sup>6</sup> 📥 🔺
▶ ♥ ●	■ ♥ ☆	▶ ♥ ●
	$\blacklozenge \bullet \diamondsuit$	
7	8 📥 🔺	9
	▶ ♥	$\Delta$ $\Box$
$^{10}$ $\Delta$	11 🌩 🔺	12 🌩 🔺
	■ ♥ ☆	■♥☆
	$\blacklozenge \bullet \diamondsuit$	$\blacklozenge \bullet \diamondsuit$

**RE** distributions

## RE1 – Distribution

<sup>1</sup> 1	<sup>2</sup> 0	<sup>3</sup> 1
4 1	<sup>5</sup> 1	<sup>6</sup> 1
7 0	<sup>8</sup> 1	<sup>9</sup> 0
<sup>10</sup> 0	<sup>11</sup> 1	<sup>12</sup> 1

RE5 – Distribution

1 0	<sup>2</sup> 1	<sup>3</sup> 0
4 0	<sup>5</sup> 0	<sup>6</sup> 0
7 0	8 0	<sup>9</sup> 1
<sup>10</sup> 1	<sup>11</sup> 0	<sup>12</sup> 0

RE9 – Distribution

3

6

<sup>9</sup> 0

<sup>12</sup> 1

0

0

 $^{2}$  0

<sup>5</sup> 1

8 0

11 1

0

4 0

7 0

10 0

1 0	<sup>2</sup> 0	<sup>3</sup> 1
4 1	<sup>5</sup> 1	<sup>6</sup> 1
<sup>7</sup> 0	<sup>8</sup> 1	<sup>9</sup> 0
<sup>10</sup> 0	<sup>11</sup> 1	<sup>12</sup> 1

RE2 – Distribution

RE6 – Distribution

<sup>1</sup> 0	<sup>2</sup> 1	<sup>3</sup> 0
4 0	<sup>5</sup> 0	<sup>6</sup> 0
7 1	<sup>8</sup> 0	<sup>9</sup> 1
<sup>10</sup> 0	<sup>11</sup> 0	<sup>12</sup> 0

RE10 – Distribution

 1
 0
 2
 0
 3
 0

 4 1
 5 1
 6 1

 7 0
 8 0
 9 0

 10 0
 11 1
 12 1

## RE3 – Distribution

1 0	<sup>2</sup> 0	<sup>3</sup> 0
4 0	5 1	<sup>6</sup> 0
7 0	<sup>8</sup> 0	<sup>9</sup> 0
<sup>10</sup> 0	<sup>11</sup> 1	<sup>12</sup> 1

RE7 – Distribution

1 0	<sup>2</sup> 0	<sup>3</sup> 1
4 1	<sup>5</sup> 1	<sup>6</sup> 1
7 0	<sup>8</sup> 1	<sup>9</sup> 0
<sup>10</sup> 0	<sup>11</sup> 1	<sup>12</sup> 1

1 0	<sup>2</sup> 0	<sup>3</sup> 1
4 1	<sup>5</sup> 0	<sup>6</sup> 1
7 0	<sup>8</sup> 1	<sup>9</sup> 0
<sup>10</sup> 0	<sup>11</sup> 0	<sup>12</sup> 0

## RE8 – Distribution

1 0	<sup>2</sup> 0	<sup>3</sup> 0
4 0	5 1	<sup>6</sup> 0
7 0	<sup>8</sup> 0	<sup>9</sup> 0
<sup>10</sup> 0	<sup>11</sup> 1	<sup>12</sup> 1

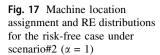
ibution	RE11 – Distribution
ibution	RETT – Distribution

1 0	<sup>2</sup> 0	<sup>3</sup> 0
4 0	<sup>5</sup> 1	<sup>6</sup> 0
7 0	8 0	<sup>9</sup> 0
<sup>10</sup> 0	<sup>11</sup> 1	<sup>12</sup> 1

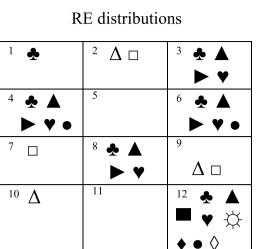
Fig. 16 RE distributions of the produced robust layout design for the risk-free case under scenario#4 ( $\alpha = 1$ )

distributions in Fig. 12, "1" shows that the relevant factory location contains this RE. Otherwise, the factory locations that don't cover the relevant RE are marked with "0". In fact, these REs distributions can be provided by using the

machine location assignment decisions. If a machine is assigned to any location, all its processing capabilities or REs will be covered by that location.



Machine locations						
1 1	<sup>2</sup> 5	<sup>3</sup> 3				
4 4	<sup>5</sup> 2	<sup>6</sup> 4				
7 6	8 3	<sup>9</sup> 5				
10 7	<sup>11</sup> <b>2</b>	12 2				



RE1 – Distribution RE2 – Distribution			tion	RE3 – Distribution				RE4 – Distribution				
<sup>1</sup> 1	<sup>2</sup> 0	<sup>3</sup> 1	1 0	<sup>2</sup> 0	<sup>3</sup> 1	1 0	<sup>2</sup> 0	<sup>3</sup> 0		1 0	<sup>2</sup> 0	3 1
4 1	5 <u>0</u>	<sup>6</sup> 1	4 1	<sup>5</sup> <u>0</u>	<sup>6</sup> 1	4 0	<sup>5</sup> <u>0</u>	<sup>6</sup> 0		4 1	<sup>5</sup> <u>0</u>	<sup>6</sup> 1
<sup>7</sup> 0	<sup>8</sup> 1	<sup>9</sup> 0	7 0	<sup>8</sup> 1	<sup>9</sup> 0	7 0	<sup>8</sup> 0	<sup>9</sup> 0	Ē	<sup>7</sup> 0	<sup>8</sup> 1	<sup>9</sup> 0
<sup>10</sup> 0	<sup>11</sup> <u>0</u>	<sup>12</sup> 1	<sup>10</sup> 0	<sup>11</sup> <u>0</u>	<sup>12</sup> 1	<sup>10</sup> 0	<sup>11</sup> <u>0</u>	12 1	-	<sup>10</sup> 0	<sup>11</sup> <b>0</b>	<sup>12</sup> 0
RE5-	– Distrib	ution	RE6 –	Distribu	ution	 RE7 -	- Distrib	ution		RE8 -	– Distrib	oution
<sup>1</sup> 0	<sup>2</sup> 1	<sup>3</sup> 0	1 0	<sup>2</sup> 1	<sup>3</sup> 0	1 0	<sup>2</sup> 0	<sup>3</sup> 1		1 0	<sup>2</sup> 0	3 0
4 0	<sup>5</sup> <u>0</u>	<sup>6</sup> 0	4 0	<sup>5</sup> <u>0</u>	<sup>6</sup> 0	4 1	<sup>5</sup> <u>0</u>	<sup>6</sup> 1	-	4 0	<sup>5</sup> <u>0</u>	6 0
<sup>7</sup> 0	<sup>8</sup> 0	<sup>9</sup> 1	7 1	<sup>8</sup> 0	<sup>9</sup> 1	7 0	<sup>8</sup> 1	<sup>9</sup> 0	-	<sup>7</sup> 0	<sup>8</sup> 0	9 0
<sup>10</sup> 1	<sup>11</sup> <u>0</u>	<sup>12</sup> 0	<sup>10</sup> 0	<sup>11</sup> <u>0</u>	<sup>12</sup> 0	<sup>10</sup> 0	<sup>11</sup> <u>0</u>	<sup>12</sup> 1	-	<sup>10</sup> 0	<sup>11</sup> <b>0</b>	<sup>12</sup> 1
RE9	– Distrib	oution	RE10	– Distri	bution	RE11	– Distri	bution				
1 0	<sup>2</sup> 0	<sup>3</sup> 0	1 0	<sup>2</sup> 0	<sup>3</sup> 0	1 0	<sup>2</sup> 0	<sup>3</sup> 0				
4 0	<sup>5</sup> <u>0</u>	<sup>6</sup> 0	4 1	<sup>5</sup> <u>0</u>	<sup>6</sup> 1	4 0	<sup>5</sup> <u>0</u>	<sup>6</sup> 0				
<sup>7</sup> 0	<sup>8</sup> 0	<sup>9</sup> 0	7 0	<sup>8</sup> 0	<sup>9</sup> 0	7 0	<sup>8</sup> 0	<sup>9</sup> 0				
<sup>10</sup> 0	<sup>11</sup> <u>0</u>	12 1	<sup>10</sup> 0	<sup>11</sup> <u>0</u>	12 1	<sup>10</sup> 0	<sup>11</sup> <u>0</u>	<sup>12</sup> 1				

Fig. 18 RE distributions of the produced robust layout design for the risk-free case under scenario#2 ( $\alpha = 1$ )

In a similar way, the produced robust capability-based distributed layout design and its relevant RE distributions are also depicted in Figs. 13, 14 under scenario#2, where M2 and M3 in MHP Machining Center-1 are broken down. Since machines M2 and M3 in MHP Machining Center-1

are broken down under scenario#2, their locations don't contain their processing capabilities as shown in Fig. 13. Their REs are also marked with "0" for these locations in the RE distribution tables (see Fig. 14). When the layout score of the robust layout design under this scenario#2 is compared to the scenario#4, it is obvious that its layout score will get worse since the unavailable machines lead to this unsatisfied scenario. In other words, total part flow rates as well as the total distance from the unoccupied locations to the occupied ones by the REs will increase when these critical machines are broken down.

Finally, the machine location assignments and the RE distribution tables of the generated risk-free robust layout design alternatives ( $\alpha = 1$ ) are also displayed in Figs. 15, 16, 17, 18 under both scenario#4 and scenario#2. In contrast with the most risky case ( $\alpha = 0$ ), the feasibility degree of the fuzzy-stochastic constraint set in Eq. (6) will be 100%, and therefore, this robust layout design option in Figs. 15, 16, 17 can be stated as a fully acceptable one, which is generally desired by risk-averse facility designers. However, there is still one unsatisfied scenario for this robust layout design option because of the random machine breakdowns under scenario#2. Fortunately, if the probability values for the machine availabilities that are formerly given in Table 15 are increased by performing additional preventive maintenance activities, the number of unsatisfied scenarios for these chance constraint sets will decrease.

For that reason, a risk-averse facility designer should focus more on accurate demand forecasting and machine

failure prevention systems to obtain risk-free robust layout design options under various scenarios and uncertainty levels.

### 6.3 Comparison of results with the existing cellular layout of the manufacturing company

Lastly, all of the generated solutions by the proposed robust biased CBDL approach are compared to the existing cellular layout of the manufacturing company as given in Table 19 and Fig. 19. In order to make a proper comparison, the machine location assignment in the previously given existing cellular layout design (see Fig. 10) is utilized while running the proposed fuzzy-stochastic programming model. When the results of the existing cellular layout are examined in detail, it is recognized that the same machines, i.e., M2 and M3 in MHP Machining Center-1, are also critical for the existing cellular manufacturing system since the unavailability of these machines might cause violation of the chance constraint set under scenario#2. It is obviously seen from the comparative results in Table 19 and Fig. 19 that considerable improvements in the total expected layout score can be achieved through the

Table 19Comparison of theexisting cellular layout with the		Total layout scores (Objective values)					
proposed R-CBDL design approach		The existing cellular layout	The proposed robust CBDL design approach				
approach	The most risky case $(\alpha = 0)$	180.554	144.866				
	The risk-free case ( $\alpha = 1$ )	181.508	145.7975				

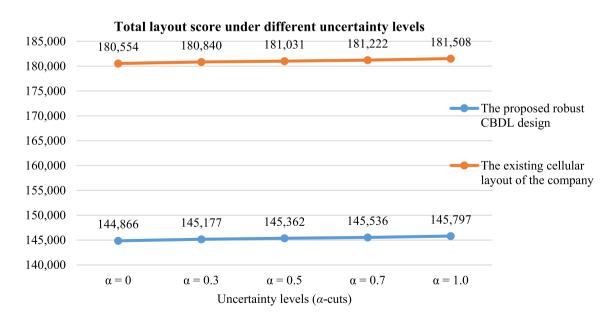


Fig. 19 Comparison of the scores of the proposed layouts with the existing cellular layout under different uncertainty levels

proposed R-CBDL approach. Indeed, the proposed approach can provide considerable decreases in the total part flows among different machines for each uncertainty level. Moreover, each processing capability can be reached from every location of the facility floor with the minimum total distance despite the random machine breakdowns. Therefore, more effective and efficient material handling operations can be yielded by using the proposed robust biased CBDL approach in case of uncertain demand data and random machine breakdowns.

# 7 Conclusions, discussions, and future research directions

In the present paper, a novel robust capability-based distributed layout (R-CBDL) design problem, which can handle random machine breakdowns (or machine unavailability risk) and fuzzy demand/process flow information, is introduced for the first time in the literature. First, a new fuzzy-stochastic optimization model of this problem is formulated based on the original deterministic MILP model of Baykasoğlu and Subulan (2020). Then, a hybrid solution approach based on a chance-constrained stochastic program and an interactive fuzzy resolution method is also proposed to transform the fuzzy-stochastic optimization model into its deterministic equivalent form. The proposed approach can address different types of uncertainties concurrently such as randomness and fuzziness and can also generate various layout design alternatives under different probabilistic scenarios and uncertainty levels concerning the facility designer's risk attitude (i.e., risk-averse or seeker).

The extensive computational experiments based on both an illustrative example and a real-life application study have revealed the following key findings, conclusions, and managerial insights to the facility designers: (i) According to the deterministic, fuzzy, stochastic, and fuzzy-stochastic optimization results, it is inferred that the layout score that is composed of total distances and part flow rates will decrease when the machine capability overlap is increased. Therefore, the degree of machine capability overlap has a substantial effect on the layout objective under both crisp and uncertain environments. (ii) The random machine breakdown has also a considerable effect on the layout objective. Indeed, when the deterministic and stochastic optimization results are compared, it is inferred that the layout score will deteriorate in case of random breakdowns (or unavailability) of critical machines. In other words, total part flow rates as well as the distances (i.e., the total expected score of this robust layout design) from the unoccupied locations to the occupied ones by the machining capabilities will increase when some of the critical machines are unavailable or broken down. Fortunately, the proposed hybrid solution approach is able to specify these critical machines (with the most vital processing capabilities) by using the unsatisfied chance constraint sets. In fact, the simultaneous breakdown of critical machines may lead to unsatisfied scenarios, whereas the non-critical machines don't cause violation of any chance constraint since their alternative machines that have the same processing capabilities are available or not broken down. Consequently, facility managers and designers can be aware of these critical machines and aim to increase their reliability rates (or decrease their breakdown probabilities) to maintain the continuous production of the facilities. Thus, more reliable and robust layout design alternatives are obtained via the proposed approach under different probabilistic scenarios and uncertainty levels. (iii) For the risk-averse facility designers, the proposed approach generated a risk-free robust layout design alternative in which "the feasibility degree of fuzzy-stochastic constraints is high ( $\alpha = 1$ ), whereas the number of unsatisfied scenarios for the chance constraints is low." If the machine availabilities are increased by performing additional preventive maintenance activities, the number of unsatisfied scenarios for the chance constraint sets will decrease. For that reason, risk-averse facility designers should focus more on accurate demand forecasting and machine failure prevention systems to obtain risk-free robust layout designs. (iv) For a risk-seeking facility designer, the proposed approach produced a risky layout design alternative with a comparatively better layout score. However, this layout design alternative will have a low feasibility degree ( $\alpha = 0$ ) for the fuzzy-stochastic constraints and also a large number of unsatisfied scenarios for the chance constraint sets. It should be emphasized here that higher feasibility degrees and satisfaction probabilities for the fuzzy and chance constraint sets are generally desired by facility planners in real-life applications. (v) The uncertainty in the part demands has not a crucial influence on the layout objective (or score) as much as the random machine breakdowns. On the other hand, different uncertainty levels ( $\alpha$ -cuts) of the fuzzy part flows may change the facility layout design completely. This is the reason why we took into account the fuzzy part flow rates with such an  $\alpha$ -parametric interactive fuzzy resolution approach. (vi) The comparative results of the real-life application study have also demonstrated that considerable improvement (i.e., on average 24.5%) on the total expected layout score can be achieved by using the proposed approach when compared to the existing cellular layout of the manufacturing company. In detail, it can provide considerable decreases in the total part flows among different machine locations for each uncertainty level. Therefore, each machining capability can be reached from every location of the facility floor with minimum total distance by considering the random machine breakdowns. Hence, more efficient material handling operations can be acquired via the proposed approach in case of uncertain demand, process flows, and random machine breakdowns.

In the future, a matheuristic solution approach that hybridizes a metaheuristic algorithm with the proposed fuzzy-stochastic mathematical programming model can be developed to solve the larger-sized problem instances. Moreover, the proposed R-CBDL problem studied in this paper has just considered equal machine sizes (or department areas). Hence, the development of a fuzzy-stochastic optimization model for an unequal-area R-CBDL design problem can also be scheduled as future work. Furthermore, the hybrid uncertainty (or fuzzy random variables) proposed by Liu (2007) that contains both fuzziness and randomness inherently can also be used to deal with nondeterministic layout design parameters in future research.

Author contributions K.S. conceived the study, participated in the design of the solution algorithms and implementation of the mathematical models, carried out computational experiments, analyzed experimental results, and wrote the paper with the other authors. B.V. participated in the design of the study and initial versions of the manuscript and analyzed the computational results. A.B. participated throughout the preparation of the paper. All authors read and approved the final manuscript.

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Data availability Data are available on request from the authors.

**Code availability** In case of this paper is accepted, the data and codes needed for replication of our results will be available on request from the authors.

#### Declarations

**Conflict of interest** Authors have no conflicts of interest to declare that are relevant to the content of this article.

**Ethical approval** No human participants or animals have been involved in this study.

Informed consent Not applicable.

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